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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 56, PART 6 1 November 1944 No. 318

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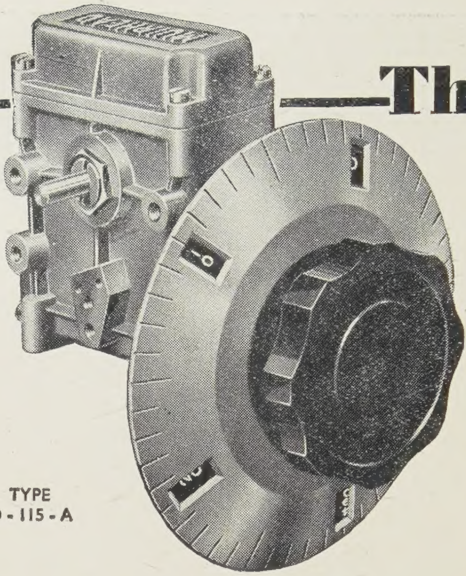
Price to non-Fellows 8*s*. 4*d*. net; post free 8*s*. 9*d*.
Annual subscription 42*s*. post free, payable in advance

Published by
THE PHYSICAL SOCIETY
1 Lowther Gardens, Exhibition Road, London S.W.7

Printed by
TAYLOR AND FRANCIS, LTD.,
Red Lion Court, Fleet Street, London E.C.4



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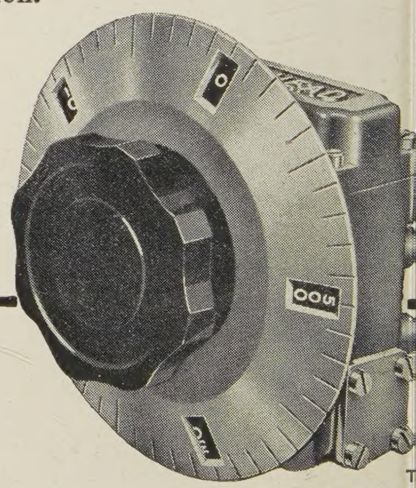

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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 56, PART 6 1 November 1944 No. 318

AN APPROXIMATE METHOD FOR CALCULATING HEAT FLOW IN AN INFINITE MEDIUM HEATED BY A CYLINDER

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MS. received 9 *May* 1944

ABSTRACT. An approximate equation is given for the solution of the rapidly heated or naturally cooled long highly-conducting cylinder in a conducting medium. The range of validity and accuracy of the approximation are indicated. The analogy with a loaded transmission line is shown, by which is illustrated the different types of solution according to the magnitude of the ratio of the effective specific heat of the cylinder to the specific heat of the medium. Some formulae for the rapid computation over certain ranges of the error integral with a complex argument are given.

§1. ORIGIN OF THE PROBLEM

In engineering practice it often arises that a long cylinder of high thermal conductivity (e.g. metallic) is rapidly heated and then cools by conduction into the surrounding medium. Examples are the heating of an electric cable by a short-circuit current or the heating of a pipe by the condensation of steam or other vapour. Interest lies usually either in the conduction of heat during the heating period (in order to provide a correction for the usual assumption that the temperature is given by the quotient of the heat supplied by the thermal capacity of the cylinder) or else in the rate of cooling of the cylinder after removal of the source of heat, for the short period before it becomes more convenient and wiser to measure the temperatures rather than to rely upon the calculation of an idealized case. A sufficiently accurate formula for long times for a cylinder in a semi-infinite medium has been given elsewhere, for cable operation, by the present author (1938). The cases here under consideration refer to periods short enough for the heat flow to be uninfluenced by the outer boundary conditions even in an example such as a single-conductor cable, where the thickness of the dielectric is of the order of the radius of the heated conductor. In mathematical terms a solution is desired of the equation

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=\frac{1}{D}\frac{\partial\theta}{\partial t}=\frac{\partial\theta}{\partial\tau}, \qquad \ldots\ldots(1.1)$$

where θ = temperature rise at an axial distance r in the medium, t = time, D = diffusivity of the medium, $\tau=Dt$, while $\theta=0$ when $r=\infty$ and $\theta=0$ when $t=0$ throughout the medium. During the heating period

$$\left(\frac{C\partial\theta}{\partial t}\right)_{r=a}=H+2\pi ak\left(\frac{\partial\theta}{\partial r}\right)_{r=a}, \qquad \ldots\ldots(1.1.1)$$

where a = radius of heated cylinder, k = thermal conductivity of medium, C = thermal capacity of cylinder per unit length and H = rate at which energy is developed in the cylinder. During the cooling period,

$$\theta(r=a,\ t=0)=\theta_0, \quad C\left(\frac{\partial\theta}{\partial t}\right)_{r=a}=2\pi ka\left(\frac{\partial\theta}{\partial r}\right)_{r=a}. \qquad \text{......(1.1.2)}$$

This neglects the temperature distribution in the medium due to conduction in the heating period, but this can be applied as a correction since it is both small and, the first problem having been solved, deducible by known formulae.

Let $\bar\theta$ = Laplace transform of θ with respect to τ, then

$$\frac{d^2\bar\theta}{dr^2}+\frac{1}{r}\frac{d\bar\theta}{dr}-p\bar\theta=0, \qquad \text{......(1.2)}$$

with $\bar\theta=0$ when $r=\infty$ and $\bar\theta=0$ when $\tau=0$. During the heating period

$$DCp\bar\theta=H+2\pi ka(\partial\bar\theta/\partial r) \text{ at } r=a. \qquad \text{......(1.2.1)}$$

During the cooling period

$$DCp\bar\theta-DC\theta_0=2\pi ka(\partial\bar\theta/\partial r) \text{ at } r=a. \qquad \text{......(1.2.2)}$$

Even if the simplifying assumption is made in (1.1.1) for the heating period that at $r=a$

$$\theta=\theta_c=Ht, \qquad \text{......(1.1.3)}$$

the solutions are complicated and give, for the heating period,

$$\bar\theta=(H/Dp^2)\,K_0(r\sqrt{p})/K_0(a\sqrt{p}),$$
$$\theta=\frac{H}{2\pi Dj}\int_{c-j}^{c+j\infty}\frac{e^{p\tau}}{p^2}\frac{K_0(r\sqrt{p})}{K_0(a\sqrt{p})}\,dp, \qquad \text{......(1.3.1)}$$

and for the cooling period,

$$\bar\theta/\theta_0=K_0(r\sqrt{p})/\{p\,K_0(a\sqrt{p})-(2s'\sqrt{p}/sa)\,K_0'(a\sqrt{p})\}$$
$$\theta/\theta_0=(1/2\pi j)\int_{c-j\infty}^{c+j\infty}K_0(r\sqrt{p})e^{p\tau}\,dp/\{pK_0(a\sqrt{p})-(2s'\sqrt{p}/sa)K_0'(a\sqrt{p})\}, \qquad \text{......(1.3.2)}$$

where s = effective specific heat of cylinder, s' = specific heat of medium, and K_0 represents a Bessel function of the second kind, zero order and purely imaginary argument as introduced by Gray and Mathews. In principle these solutions can be evaluated as contour integrals; equation (1.3.2) requires a knowledge of the roots of $sa\sqrt{p}\,K_0(a\sqrt{p})=2s'K_0'(a\sqrt{p})$. The method is adequately described by Jaeger (1944) for a closely analogous problem in an article published after the present paper had reached its substantially final form, but in the present instance the application is somewhat laborious. In two recent papers Grunberg and Grunberg and Soutz (1941) have indicated a useful method of arriving at a first approximation to the problem of the heating period, obtaining

$$\theta=\frac{H}{C(z_1-z_2)}\sqrt{\left(\frac{a}{r}\right)}\left[\frac{1}{z_1}\left\{e^{z_1^2t-(xz_1/\sqrt{D})}\left(1-\Phi\left(\frac{x}{2\sqrt{Dt}}-z_1\sqrt{t}\right)\right)-\left(1-\Phi\left(\frac{x}{2\sqrt{Dt}}\right)\right)\right\}\right.$$
$$\left.-\frac{1}{z_2}\left\{e^{z_2^2t-(xz_2/\sqrt{D})}\left(1-\Phi\left(\frac{x}{2\sqrt{Dt}}-z_2\sqrt{t}\right)\right)-\left(1-\Phi\left(\frac{x}{2\sqrt{Dt}}\right)\right)\right\}\right], \qquad \text{......(1.4)}$$

where $x=r-a$, while z_1 and z_2 are the roots of the equation

$$Cz^2-2as'z\sqrt{D}+\pi k-H\alpha=0 \qquad \ldots\ldots(1.4.1)$$

and

$$\Phi(y)=\frac{2}{\sqrt{\pi}}\int_0^y e^{-v^2}\,dy;$$

H = initial rate of heat generation; α represents the coefficient of H with temperature (as may happen, due to a temperature coefficient of resistance); it has hitherto been taken as zero.

Higher-order approximations are given by the authors mentioned, who use indeed a method essentially similar to that of Jaeger (loc. cit.), but it is here intended only to apply the first approximation method to the problem of the cooling period. This presents more difficulty than the heating period, but the method gives a readily computable result, satisfactory for many applications, while at the same time it shows some features of interest by analogy with the variably loaded "Bessel" transmission line.

§ 2. THE APPROXIMATE EQUATIONS FOR THE COOLING PERIOD

In equation (1.1) substitute $u=\theta\sqrt{(r/a)}$, obtaining

$$\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial x^2}+\frac{u}{4(a+x)^2}, \text{ if } x=r-a. \qquad \ldots\ldots(2.1)$$

If interest lies only in short times, the material values of x will be small, so that $u/4(a+x)^2$ may be neglected. To obtain an idea of the degree of approximation, consider the case when the cylinder is maintained at the initial temperature θ_0, so that

$$\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial x^2}, \quad u=0 \text{ when } \tau=0 \text{ or } x=\infty, \qquad \ldots\ldots(2.1.1)$$

and

$$u(x=0)=\theta_0.$$

The solution for this is known (e.g. Carslaw (1921), p. 47) and is

$$u_1/\theta_0=1-\Phi(x/2\sqrt{\tau}). \qquad \ldots\ldots(2.1.2)$$

The heat loss from the cylinder per cm.² is $k\theta_0/\sqrt{\pi\tau}$. The neglected term $u/4(a+x)^2$ may be regarded as giving rise to a loss per cm.² of

$$\int_0^\infty ku\,dx/4(a+x)^2<\int_0^\infty ku\,dx/4a^2=(k\theta_0/2a^2)\sqrt{(\tau/\pi)}. \qquad \ldots\ldots(2.1.3)$$

The error in neglecting the term will not exceed the ratio of the heat loss arising from the neglected term to the rough value of the total heat loss. This ratio is $\tau/2a^2$ or $Dt/2a^2$, which gives a rough measure of the range within which the approximation is valid.

Now find the solution to $\partial u/\partial\tau=\partial^2 u/\partial x^2$, when $u=0$ at $x=\infty$, $u=\theta_0$ when $x=0$ and $\tau=0$, $u=0$ when $\tau=0$ and

$$C\frac{\partial u}{\partial t}=2\pi ka\frac{\partial}{\partial r}\left(u\sqrt{\frac{a}{r}}\right) \text{ at } r=a. \qquad \ldots\ldots(2.2.1)$$

Let $\bar{u}$ = Laplace transform of u with respect to τ, then

$$\frac{d^2\bar{u}}{dx^2} = p\bar{u}, \qquad \text{......(2.3)}$$

$\bar{u}=0$ at $x=\infty$, $\bar{u}=0$ when $\tau=0$, and

$$DC\theta_0 - DCp\bar{u} = \pi k\{\bar{u} - 2a(d\bar{u}/dx)\} \text{ at } x=0. \qquad \text{......(2.3.1)}$$

The solution is

$$\bar{u} = \frac{DC\theta_0 e^{-x\sqrt{p}}}{DCp + 2\pi ka\sqrt{p} + \pi k} = \frac{\theta_0 e^{-x\sqrt{p}}}{p + (2\lambda/a)\sqrt{p} + (\lambda/a^2)}, \qquad \text{......(2.3.2)}$$

where $\lambda = s'/s$. It now appears that the roots of the denominator are real, coincident or complex according as the specific heat of the medium is greater than, equal to, or less than the effective specific heat of the cylinder. The physical significance of this is conveniently appreciated by considering the "Bessel" line or long transmission line from the end. If now an end length of the line is replaced by a capacitance less than its equivalent capacitance, which is suddenly charged, the charge will decay according to a line artificially shortened according to the capacitance defect (specific heat of cylinder less than that of medium). If replaced by its exact equivalent capacitance, the charge will decay according to the natural relaxation of the line (specific heats equal). If, however, the replacing capacitance exceeds the equivalent capacitance of the length removed, the situation cannot correspond to any equivalent length of line, because the law of the line does not provide for its artificial extension beyond the point at which the admittance becomes zero, so that a species of reflexion will occur (specific heat of cylinder greater than that of medium). Accordingly the three cases must be distinguished because the solutions will differ essentially.

In practice the case $s' \geqslant s$ will occur with metal pipes of medium wall thickness and with hollow-core cables. The case $s' < s$ will occur with solid cylinders, thick-walled pipes or cable conductors of the usual stranded type.

§ 3. THE CASE $s' > s$

Applying the inverse transform integral and decomposing the integrand into partial fractions,

$$u = \frac{\theta_0}{2}\sqrt{\left(\frac{a^2s^2}{s(s'-s)}\right)}\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} e^{p\tau - x\sqrt{p}}\left(\frac{1}{\sqrt{p}-z_1} - \frac{1}{\sqrt{p}-z_2}\right)dp, \qquad \text{......(3.1)}$$

where

$$\left.\begin{aligned} z_1 &= -(s'/sa) + \sqrt{\{s'(s'-s)/s^2a^2\}} \\ z_2 &= -(s'/sa) - \sqrt{\{s'(s'-s)/s^2a^2\}} \end{aligned}\right\} \qquad \text{......(3.1.1)}$$

The integral (3.1) may be evaluated by transformation of the independent variable to $\sqrt{p} - z$ or v (say). Any power of v in the denominator of a resulting term may be removed by integration by parts, while if v occurs in the denominator it may be removed by differentiation with respect to x under the integral sign. It is then easy to transform the exponential term to the form e^{v^2} so that the integration from $j\infty$ to $-j\infty$ can be determined, leaving only, in some cases, an integration with respect to x. By this means the following answer may be obtained:

$$\frac{u}{\theta_0}=\frac{1}{2\epsilon}\left[(1+\epsilon)\left\{1-\Phi\left(\frac{x}{2\sqrt{Dt}}+\frac{s'}{s}\frac{\sqrt{Dt}}{a}+\frac{\epsilon s'}{as}\sqrt{Dt}\right)\right\}\exp\left\{\frac{s'x}{sa}+\frac{x\epsilon}{a}\frac{s'}{s}\right.\right.$$
$$\left.+\frac{Dt}{a^2}\left(\frac{2s'^2}{s^2}-\frac{s'}{s}+\frac{2s'^2\epsilon}{s^2}\right)\right\}$$
$$-(1-\epsilon)\left\{1-\Phi\left(\frac{x}{2\sqrt{Dt}}+\frac{s'}{s}\frac{\sqrt{Dt}}{a}-\frac{\epsilon s'}{as}\sqrt{Dt}\right)\right\}$$
$$\left.\times\exp\left\{\frac{s'x}{sa}-\frac{x\epsilon}{a}\frac{s'}{s}+\frac{Dt}{a^2}\left(\frac{2s'^2}{s^2}-\frac{s'}{s}-\frac{2s'^2\epsilon}{s^2}\right)\right\}\right], \qquad \ldots\ldots(3.2.1)$$

where $\epsilon=\sqrt{\dfrac{s'-s}{s'}}$.

Let $\xi^2=Dt/a^2$, $\eta=x/a$, $\lambda=s'/s$,

$$\frac{\theta}{\theta_0}=\frac{1}{2}\sqrt{\frac{\lambda a}{(\lambda-1)r}}\left[\left(1+\sqrt{1-\frac{1}{\lambda}}\right)\left\{1-\Phi\left(\frac{\eta}{2\xi}+\xi(\lambda+\sqrt{\lambda^2-\lambda})\right)\right\}\exp\{\eta(\lambda+\sqrt{\lambda^2-\lambda})\right.$$
$$+\xi^2(2\lambda^2-\lambda+2\lambda\sqrt{\lambda^2-\lambda})\}-\left(1-\sqrt{1-\frac{1}{\lambda}}\right)\left\{1-\Phi\left(\frac{\eta}{2\xi}+\xi(\lambda-\sqrt{\lambda^2-\lambda})\right)\right\}$$
$$\left.\exp\{\eta(\lambda-\sqrt{\lambda^2-\lambda})+\xi^2(2\lambda^2-\lambda-2\lambda\sqrt{\lambda^2-\lambda})\}\right]. \qquad \ldots\ldots(3.2.2)$$

The temperature of the cylinder θ_c is given by

$$\frac{\theta_c}{\theta_0}=\frac{1}{2}\sqrt{\frac{\lambda}{\lambda-1}}\left[\left(1+\sqrt{1-\frac{1}{\lambda}}\right)\{1-\Phi(\xi(\lambda+\sqrt{\lambda^2-\lambda}))\}\,e^{\lambda\xi^2(2\lambda-1+2\sqrt{\lambda^2-\lambda})}\right.$$
$$\left.-\left(1-\sqrt{1-\frac{1}{\lambda}}\right)\{1-\Phi(\xi(\lambda-\sqrt{\lambda^2-\lambda}))\}\,e^{\lambda\xi^2(2\lambda-1-2\sqrt{\lambda^2-\lambda})}\right]. \qquad \ldots\ldots(3.2.3)$$

The expressions are rapidly calculated from tables of the error integral, and particular cases, such as when λ is great, are given in textbooks. As already explained, the heat flow in this case follows a complex relaxation law derived from the sum of two simple relaxation laws.

§ 4. THE CASE $s'=s$

In this instance

$$\frac{u}{\theta_0}=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{e^{p\tau-x\sqrt{p}}}{\{\sqrt{p}+(\lambda/a)\}^2}\,dp. \qquad \ldots\ldots(4.1)$$

Put $v=\sqrt{p}+(\lambda/a)$

$$\frac{u}{\theta_0}=\frac{e^{-(\tau/a^2)+(x/a)}}{2\pi j}\left[\int_{c-j\infty}^{c+j\infty}\frac{e^{v^2\tau-(2\tau/a)+xv}}{v}\,dv+\int_{c+j\infty}^{c-j\infty}\frac{e^{v^2\tau-(2\tau/a)+xv}}{av^2}\right]dv. \qquad \ldots\ldots(4.2)$$

Integrate the second term by parts

$$\frac{u}{\theta_0}=e^{-(\tau/a^2)+(x/a)}\left[1-\Phi\left(\frac{x}{2\sqrt{\tau}}+\frac{\sqrt{\tau}}{a}\right)-\frac{\sqrt{\tau}}{a}\frac{2}{\sqrt{\pi}}\,e^{-\left(\frac{\sqrt{\tau}}{a}+\frac{x}{2\sqrt{\tau}}\right)^2}\right.$$
$$\left.+\frac{1}{a}\left(\frac{2\tau}{a}+x\right)\left\{1-\Phi\left(\frac{x}{2\sqrt{\tau}}+\frac{\sqrt{\tau}}{a}\right)\right\}\right] \qquad \ldots\ldots(4.2.1)$$

$$\frac{u}{\theta_0}=e^{-\xi^2+\eta}\left[(1+2\xi^2+\eta)\left\{1-\Phi\left(\frac{\eta}{2\xi}+\xi\right)\right\}-\frac{2\xi}{\sqrt{\pi}}e^{-\left(\frac{\eta}{2\xi}+\xi\right)^2}\right], \qquad \ldots\ldots(4.2.2)$$

$$\frac{\theta}{\theta_0}=\sqrt{\frac{a}{r}}\,e^{-\xi^2+\eta}\left[(1+2\xi^2+\eta)\left\{1-\Phi\left(\frac{\eta}{2\xi}+\xi\right)\right\}-\frac{2\xi}{\sqrt{\pi}}e^{-\left(\frac{\eta}{2\xi}+\xi\right)^2}\right] \qquad \ldots\ldots(4.3$$

and

$$\frac{\theta_c}{\theta_0}=e^{-\xi^2}\left[(1+2\xi^2)\{1-\Phi(\xi)\}-\frac{2\xi^2}{\sqrt{\pi}}e^{-\xi^2}\right]. \qquad \ldots\ldots(4.4)$$

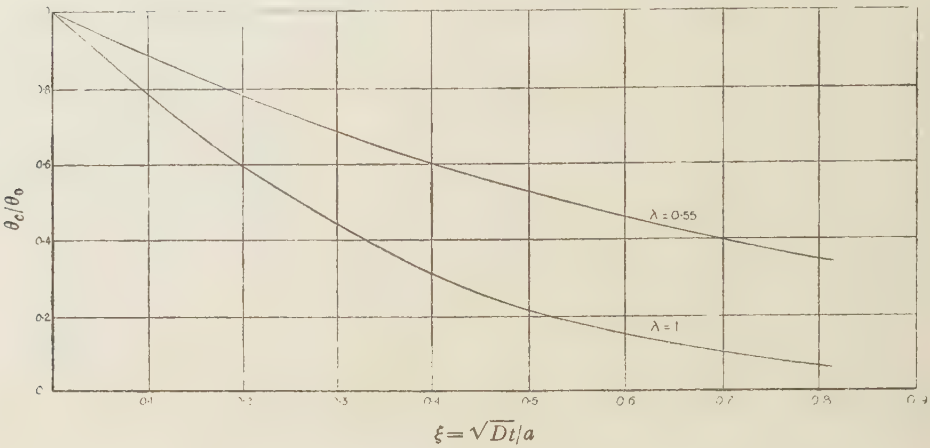

Figure 1. Cooling of cylinder.

A plot of equation (4.4) is shown in figure 1. As already explained, the heat flow in this case follows a simple relaxation law.

§ 5. THE CASE $s'<s$

Referring to equation (3.2.2), it can be seen that now

$$\frac{\theta}{\theta_0}=\sqrt{\left(\frac{\lambda a}{(1-\lambda)r}\right)}\times\frac{1}{j}\ \underset{\text{part of}}{\text{imaginary}}\left(1+j\sqrt{\frac{1}{\lambda}-1}\right)\left\{1-\Phi\left(\frac{\eta}{2\xi}+\lambda\xi+j\xi\sqrt{\lambda-\lambda^2}\right)\right\}$$
$$\exp\{\eta(\lambda+j\sqrt{\lambda-\lambda^2})+\lambda\xi^2(2\lambda^2-1+2j\sqrt{\lambda-\lambda^2})\}. \qquad \ldots\ldots(5.1)$$

Let $\cos\alpha=\sqrt{\lambda}$, $\quad w^2=(\eta^2/4\xi^2)+\lambda\xi^2+\lambda\eta$, $\quad \tan\beta=\xi\sqrt{\lambda-\lambda^2}/\{(\eta/2\xi)+\lambda^2\xi^2\}$:

$$\frac{\theta}{\theta_0}=\sqrt{\left(\frac{a}{(1-\lambda)r}\right)}\times\frac{1}{j}\ \underset{\text{part of}}{\text{imaginary}}\ e^{j\alpha}\,e^{\lambda\{\eta+(2\lambda-1)\xi^2\}}\,e^{j\sqrt{\lambda-\lambda^2}(\eta+2\lambda\xi^2)}\{1-\Phi(we^{j\beta})\} \qquad \ldots\ldots(5.2)$$

$$=\sqrt{\left(\frac{a}{(1-\lambda)r}\right)}\,e^{\lambda\{\eta+(2\lambda-1)\xi^2\}}\times\frac{1}{j}\ \underset{\text{part of}}{\text{imaginary}}\ [\cos\{\alpha+\sqrt{\lambda-\lambda^2}(\eta+2\lambda\xi^2)\}$$
$$+j\sin\{\alpha+\sqrt{\lambda-\lambda^2}(\eta+2\lambda\xi)\}]$$
$$\times\left[1-\frac{2}{\sqrt{\pi}}\left\{w\cos\beta-\frac{1}{3}w^3\cos3\beta+\frac{1}{5.2!}w^5\cos5\beta\ldots-jw\sin\beta\right.\right.$$
$$\left.\left.+\frac{1}{3}jw^3\sin3\beta-\frac{1}{5.2!}jw^5\sin5\beta\ldots\right\}\right]. \qquad \ldots\ldots(5.2.1)$$

$$\frac{\theta}{\theta_0}=\sqrt{\left(\frac{a}{(1-\lambda)r}\right)}\,e^{\lambda\{\eta+(2\lambda-1)\xi^2\}}\left[\sin\left\{\alpha+\frac{1}{2}\eta\sin 2\alpha+\lambda\xi^2\sin 2\alpha\right\}\right.$$
$$\times\left\{1-\frac{2}{\sqrt{\pi}}w\cos\beta+\frac{2}{3\sqrt{\pi}}w^3\cos 3\beta\ldots\right\}$$
$$\left.-\frac{2}{\sqrt{\pi}}\cos\left\{\alpha+\frac{1}{2}\eta\sin 2\alpha+\lambda\xi^2\sin 2\alpha\right\}\left\{w\sin\beta-\frac{1}{3}w^3\sin 3\beta\ldots\right\}\right], \qquad \ldots\ldots(5.2.2)$$

when $x=0$, $r=a$, $x=0$, $\eta=0$, $\beta=\alpha$, $w=\xi\cos\alpha$ and $\theta=\theta_c$.

$$\frac{\theta_c}{\theta_0}=\sqrt{\left(\frac{1}{1-\lambda}\right)}e^{\lambda(2\lambda-1)\xi^2}\left[\sin(\alpha+\lambda\xi^2\sin 2\alpha)\right.$$
$$\times\left\{1-\frac{2}{\sqrt{\pi}}\xi\cos^2\alpha+\frac{2}{3\sqrt{\pi}}\xi^3\cos^3\alpha\cos 3\alpha\ldots\right\}$$
$$\left.-\frac{2}{\sqrt{\pi}}\cos(\alpha+\lambda\xi^2\sin 2\alpha)\left\{\xi\cos\alpha\sin\alpha-\frac{1}{3}\xi^3\cos^3\alpha\sin 3\alpha\ldots\right\}\right]. \qquad \ldots\ldots(5.3)$$

It is clear from section (2) that the present treatment is only valid when ξ is fairly small compared with unity, so the series converge rapidly and the formulae are easy to compute. Simpler formulae may, however, be obtained by expressing the error integral in terms of the confluent hypergeometric function.

Using Whittaker's notation (see Whittaker and Watson, *Modern Analysis*, 4th ed., 1927, p. 341) * for the hypergeometric functions,

$$1-\Phi(y)=\frac{2}{\sqrt{\pi}}\operatorname{Erfc}(y)=\frac{1}{\sqrt{\pi}}\frac{1}{\sqrt{y}}e^{-\frac{1}{2}y^2}W_{-\frac{1}{4},\frac{1}{4}}(y^2). \qquad \ldots\ldots(5.4.1)$$

It then follows, for example by using Kummer's expansion of the allied function $M_{k,m}(y)$, that

$$\Phi(y)=\frac{2y}{\sqrt{\pi}}e^{-y^2}\left\{1+\frac{2}{3}y^2+\frac{2^2}{3.5}y^4+\frac{2^3}{3.5.7}y^6+\ldots\right\} \qquad \ldots\ldots(5.4.2)$$

and

$$\Phi(we^{j\beta})=\frac{2w}{\sqrt{\pi}}e^{-w^2\cos 2\beta}e^{j(\beta-w^2\sin 2\beta)}\left\{1+\frac{2}{3}w^2e^{2j\beta}+\frac{2^2}{3.5}w^4e^{4j\beta}+\ldots\right\}. \qquad \ldots\ldots(5.4.3)$$

Referring to equation (5.2), this may be written in the form

$$\frac{\theta}{\theta_0}=\sqrt{\frac{a}{(1-\lambda)r}}\times\frac{1}{j}\begin{array}{c}\text{imaginary}\\ \text{part of}\end{array}e^{w^2\cos 2\beta-\frac{\eta^2}{4\xi^2}}e^{jw^2\sin 2\beta-j\alpha}\{1-\Phi(we^{j\beta})\} \qquad \ldots\ldots(5.5.1)$$

$$=\sqrt{\frac{a}{(1-\lambda)r}}\times\frac{1}{j}\begin{array}{c}\text{imaginary}\\ \text{part of}\end{array}\left[e^{w^2\cos 2\beta-\frac{\eta^2}{4\xi^2}}e^{jw^2\sin 2\beta-j\alpha}-\frac{2w}{\sqrt{\pi}}e^{-\frac{\eta^2}{4\xi^2}}e^{j(\alpha+\beta)}\right.$$
$$\left.\times\left\{1+\frac{2}{3}w^2e^{2j\beta}+\frac{2^2}{3.5}w^4e^{4j\beta}+\ldots\right\}\right] \qquad \ldots\ldots(5.5.2)$$

* In consulting this reference the following corrections should be noted :—

Page 340, second paragraph : " The formula for $W_{k,m}(2)$ . . . . . $k-\frac{1}{2}-m$ is *zero or* a negative number ".

Page 341, line 8 : $W_{-\frac{1}{4},\frac{1}{4}}(x^2)=x^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}\int_0^\infty\left(1+\frac{t}{x^2}\right)^{-\frac{1}{4}}e^{-t}\,dt.$

$$\frac{\theta}{\theta_0}=\sqrt{\frac{a}{(1-\lambda)r}}\,e^{-\eta^2/4\xi^2}\Big[e^{w^2\cos 2\beta}\sin(\alpha+w^2\sin 2\beta)-\frac{2}{\sqrt{\pi}}\Big\{w\sin(\alpha+\beta)$$
$$+\frac{2}{3}w^3\sin(\alpha+3\beta)+\frac{2^2}{3.5}w^5\sin(\alpha+5\beta)+\dots\Big\}\Big], \quad \dots\dots(5.5.3)$$

when $x=0$, $r=a$, $\theta=\theta_c$, etc.,

$$\frac{\theta_c}{\theta_0}=\sqrt{\frac{1}{1-\lambda}}\Big[e^{\lambda\xi^2\cos 2\alpha}\sin(\alpha+\lambda\xi^2\sin 2\alpha)-\frac{2}{\sqrt{\pi}}\Big\{\xi\cos\alpha\sin 2\alpha$$
$$+\frac{2}{3}\xi^3\cos^3\alpha\sin 4\alpha+\frac{2^2}{3.5}\xi^5\cos^5\alpha\sin 6\alpha+\dots\Big\}\Big]. \quad \dots\dots(5.6)$$

These formulae can be computed twice as rapidly as the preceding formulae, (5.2.2) and (5.3). A plot of θ_c/θ_0 for $\lambda=0.55$ is given in figure 1.

Considering equation (5.6), for simplicity, the second term regularly increases with time for small values of ξ, so that it gives rise to a simple relaxation. The first term contains the factor $\sin(\alpha+\lambda\xi^2\sin 2\alpha)$, so that it is periodic, with a period given by $\lambda\xi^2\sin 2\alpha=2n\pi$ or $t=2n\pi a^2\operatorname{cosec}2\alpha/\lambda D$. The mis-matching referred to in the analogue of the loaded transmission line thus gives rise to a periodic fluctuation due to reflexion at the termination. In the present application, any discussion of this feature is academic because the period is much longer than the time interval for which the approximation is valid, and a fluctuation of this magnitude cannot occur. In the present instance it is of more interest to note the influence of λ or the relative thermal capacity of the cylinder on its rate of cooling, as illustrated in figure 1. Since usually λ is the most difficult of the parameters to determine in practice, it will be appreciated that errors in estimating the rate of cooling very easily occur.

Although large values of ξ and w cannot enter into the present problem, there are electrical applications in which the present treatment remains valid over a much wider range. For this reason, and for the intrinsic interest of the error integral with a complex argument, an asymptotic expansion will be given:

$$1-\Phi(y)=\frac{e^{\frac{1}{2}-y^2}}{\sqrt{\pi}y}W_{-\frac{1}{4},\frac{1}{4}}(y^2)$$
$$=\frac{e^{-y^2}}{\sqrt{\pi}y}\Big\{1-\frac{1}{2y^2}+\frac{1.3}{2^2y^4}-\frac{1.3.5}{2^3y^6}+\frac{1.3.5.7}{2^4y^8}\dots\Big\}, \quad \dots\dots(5.7.1)$$

whence, from equation (5.5.1),

$$\frac{\theta}{\theta_0}=\sqrt{\left(\frac{a}{(1-\lambda)r}\right)}\times\frac{1}{j}\,\begin{matrix}\text{imaginary}\\\text{part of}\end{matrix}\,e^{-\eta^2/4\xi^2}\frac{e^{j(\alpha-\beta)}}{w\sqrt{\pi}}\Big\{1-\frac{1}{2}\frac{e^{-2j\beta}}{w^2}+\frac{1.3}{2^2}\frac{e^{-4j\beta}}{w^4}\dots\Big\}$$
$$\doteq\sqrt{\left(\frac{a}{(1-\lambda)r}\right)}\frac{e^{-\eta^2/4\xi^2}}{\sqrt{\pi}}\Big\{\frac{1}{w}\sin(\alpha-\beta)-\frac{1}{2w^3}\sin(\alpha-3\beta)+\frac{1.3}{2^2w^5}\sin(\alpha-5\beta)\dots\Big\},$$
$$\dots\dots(5.7.2)$$

and

$$\frac{\theta_c}{\theta_0}=\frac{1}{\sqrt{(1-\lambda)\pi}}\Big\{\frac{\sin 2\alpha}{2\xi^3\cos^3\alpha}-\frac{1.3}{2^2}\frac{\sin 4\alpha}{\xi^5\cos^5\alpha}\dots\Big\}. \quad \dots\dots(5.7.3)$$

Approximate curves of the error integral for complex arguments are shown in figure 2.

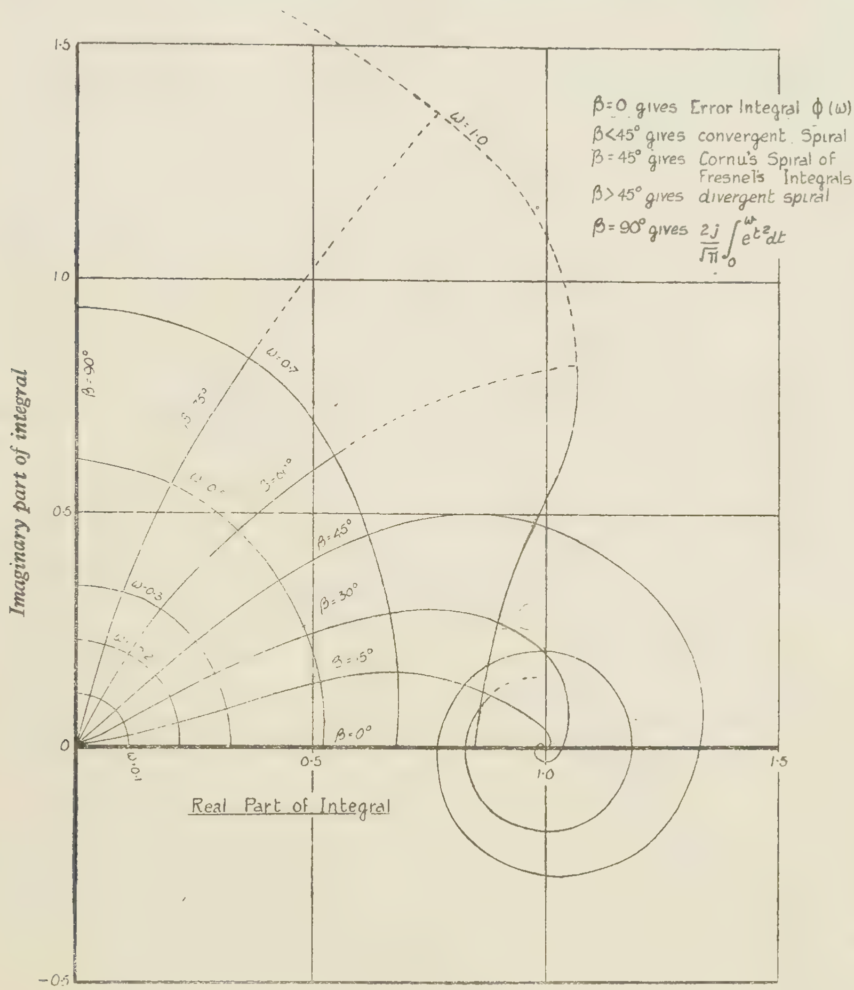

Figure 2. Argand diagram of error integral with complex argument $= \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^{\omega e^{j\beta}} e^{-t^2}\,dt.$

§ 6. CONCLUSIONS

For practical purposes wherein there is merely interest in periods during which the heat only penetrates a short distance into the medium, a first approximation to the rapid heating or the natural cooling of a cylinder (radius a) of high conductivity inserted in a conducting medium may be obtained by the use of the plane equation $\partial^2 u/\partial x^2 = \partial u/D\partial t$ when u is $\theta\sqrt{(r/a)}$, θ is the temperature at radius r, x is $r-a$ and D is the diffusivity of the medium.

The range of validity and accuracy of the approximation are given by the magnitude of $Dt/2a^2$.

The three cases when the effective specific heat of the cylinder is less than, equal to or greater than the specific heat of the medium correspond respectively

to the three cases when the input lumped capacitance to a Bessel line corresponds to an artificially shortened line, a line of natural length, or an impossibly lengthened line. The corresponding solutions are a double relaxation, a simple relaxation, or a relaxation fluctuation respectively.

The third case requires the error integral with complex arguments, formulae for the rapid computation of which are given for arguments wherein the amplitude is either less or much greater than unity.

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VIGNETTING

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ABSTRACT. The variation of effective pupil area with the obliquity to the axis of the incident light in photographic lenses depends on the position and size of the vignetting stops. This is investigated, and the possibilities outlined of choosing vignetting stops in a lens to give the pupil area variation best suited to the angular aberration distribution.

The subject of vignetting in photographic lenses, i.e. reduction of the aperture for off-axis pencils of light by lens mounts or stops in front of and/or behind the primary aperture stop, does not seem to have received much attention, though it is a very serious defect in all present-day lenses of wide aperture together with sizable angle of field. This defect is common, of course, because it has generally been found impossible to correct the higher-order coma aberrations which depend on aperture and angle of field, and hence vignetting is necessary if such lenses are to give tolerable definition. Work on the systematic correction of these higher-order comatic aberrations has necessitated the essentially simple considerations of types of vignetting which follow, since, if an improved correction falls short of a level permitting the complete elimination of vignetting, it is desirable to adopt the vignetting form best suited to the particular aberration distribution.

In most photographic lenses (other than those covering very wide angles of

field) all the glasses have their diameters limited substantially to those required to transmit the light from an axial image point at the full aperture. In this case, with an internal iris stop, the maximum possible degree of lens vignetting is used, the diameters of the front and back glasses being the effective vignetting stops, and this can only be increased by fitting external vignetting stops in the form of lens-hoods, etc. In the general case, where the vignetting stops are limited to the axial aperture diameters but are not, of course, necessarily the front and back lens diameters, vignetting will begin as soon as the object leaves the axis. The problem may be simplified by considering the (virtual) images of the vignetting stops in the object space with the object infinitely distant. In what follows, the phrase "vignetting stop" will refer to these images of the actual apertures. Then, in the case we are considering, the pupil and the front and back vignetting stops will all have the same diameter, and the effective pupil for a pencil of light inclined at an angle α to the axis will be formed by the orthogonal projection of the two vignetting apertures on the stop plane, giving an area relative to the axial aperture area as unity of

$$A=\frac{2}{\pi}\cos^{-1}\left(\frac{D}{2}\tan\alpha\right)-\frac{D\tan\alpha}{\pi}\sin\cos^{-1}\left(\frac{D}{2}\tan\alpha\right), \qquad \ldots\ldots(1)$$

where D is the separation between the front and back vignetting apertures divided by the entrance-pupil radius. It is interesting, and immediately clear

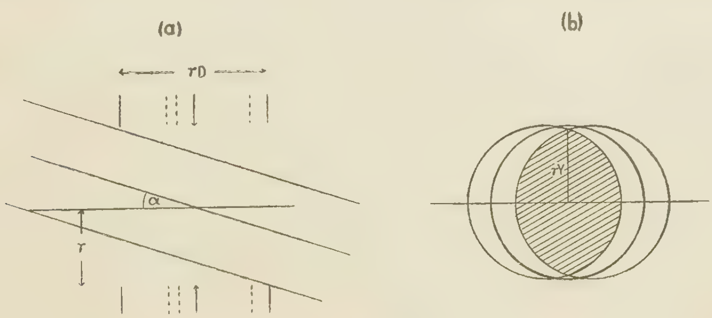

Figure 1.

from figure 1 *b*, that this is independent of the internal pupil position.

From equation (1)

$$\left.\begin{aligned}\frac{dA}{d\alpha}&=-\frac{2D}{\pi}\sec^2\alpha\,.\,Y,\\ \left(\frac{dA}{d\alpha}\right)_{\alpha=0}&=-\frac{2D}{\pi},\end{aligned}\right\} \qquad \ldots\ldots(2)$$

where Y is the distance of the intersection point of the projections of the two vignetting stops in the pupil plane from the line of centres divided by the pupil radius. As can be seen from plotting A against α (figure 2), the slope is very nearly constant until the aperture is very small, and the value of A may be taken for all practical purposes as given by the straight line

$$A=1-\frac{2D\alpha}{\pi}. \qquad \ldots\ldots(3)$$

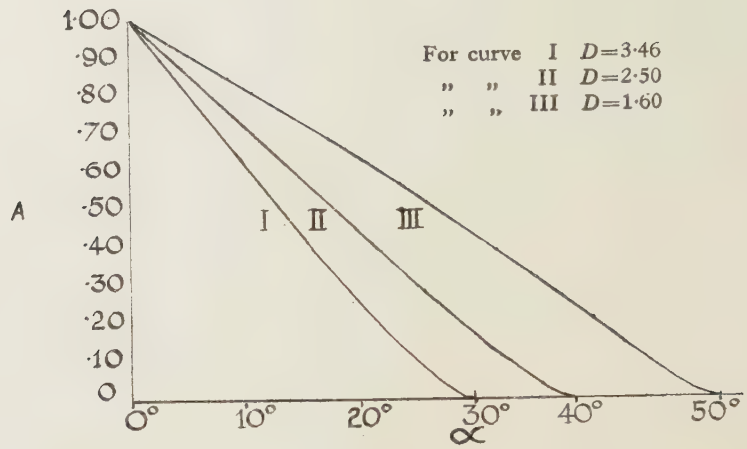

Figure 2.

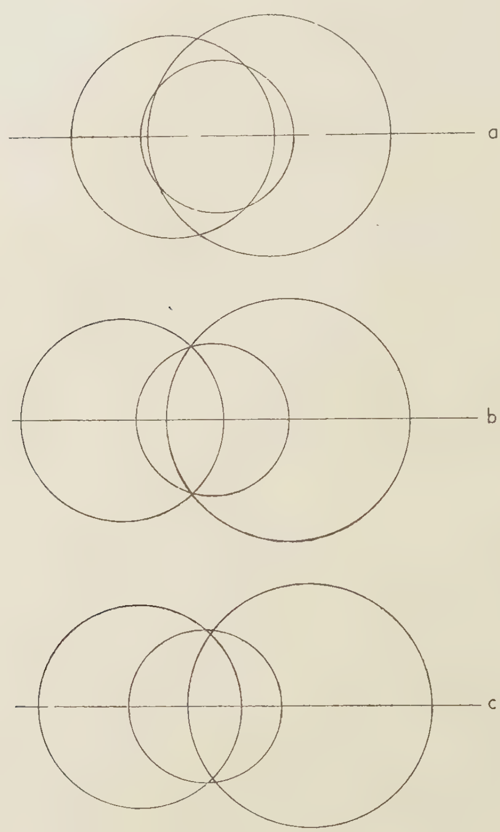

Figure 3.

In the case of single vignetting apertures in front of, and behind, the lens, each of larger radius than the entrance pupil by, say, factors R_1, R_2, and distant

from the pupil by $\frac{1}{2}D_1$, $\frac{1}{2}D_2$ of its radius, then for small angles the projections of these circles will intersect outside the pupil circle (figure 3 *a*), and the oblique aperture at angle α will be, relative to the axial aperture,

$$A=1-\frac{1}{\pi}\left[\cos^{-1}x_1-R_1^2\cos^{-1}\frac{x_1+\dfrac{D_1}{2}\tan\alpha}{R_1}\right]-\frac{D_1}{2\pi}\tan\alpha.\sin(\cos^{-1}x_1)$$

$$-\frac{1}{\pi}\left[\cos^{-1}x_2-R_2^2\cos^{-1}\frac{x_2+\dfrac{D_2}{2}\tan\alpha}{R_2}\right]-\frac{D_2}{2\pi}\tan\alpha.\sin(\cos^{-1}x_2), \quad \ldots\ldots(4)$$

where $$x_{1,2}=\frac{R_{1,2}^2-1-\dfrac{D_{1,2}^2}{4}\tan^2\alpha}{D_{1,2}\tan\alpha}.$$

This formula will apply up to that incidence angle α at which the projections of the vignetting apertures intersect on the pupil circumference (figure 3 *b*). At wider angles of field than this (figure 3 *c*)

$$\left.\begin{aligned} A&=\frac{1}{\pi}\left[R_1^2\cos^{-1}\frac{x}{R_1}+R_2^2\cos^{-1}\frac{1}{R_2}\left(\frac{D_1+D_2}{2}\tan\alpha-x\right)\right]\\ &\qquad-\frac{R_1}{\pi}.\frac{D_1+D_2}{2}\tan\alpha.\sin\left(\cos^{-1}\frac{x}{R_1}\right),\\ \text{where}\quad x&=\frac{R_2^2-R_1^2+\left(\dfrac{D_1+D_2}{2}\tan\alpha\right)^2}{2\dfrac{D_1+D_2}{2}\tan\alpha}.\end{aligned}\right\}\ldots\ldots(5)$$

Considering, for the sake of simplicity, two equal vignetting stops greater than the pupil ($R_1=R_2=R>1$) and equidistant from it ($D_1=D_2=D$), these become, for smaller angles,

$$\left.\begin{aligned} A&=1-\frac{2}{\pi}\cos^{-1}x+\frac{2R^2}{\pi}\cos^{-1}\frac{x+\dfrac{D}{2}\tan\alpha}{R}-\frac{D}{\pi}\tan\alpha.\sin(\cos^{-1}x),\\ \text{where}\quad x&=\frac{R^2-1-\left(\dfrac{D}{2}\tan\alpha\right)^2}{D\tan\alpha},\end{aligned}\right\}\ldots\ldots(6)$$

and for greater angles

$$A=\frac{2R^2}{\pi}\cos^{-1}\frac{D\tan\alpha}{2R}-\frac{R}{\pi}D\tan\alpha.\sin\left(\cos^{-1}\frac{D\tan\alpha}{2R}\right). \quad \ldots\ldots(7)$$

These two equations, (6) and (7), each give

$$\frac{dA}{d\alpha}=-\frac{2DY}{\pi}\sec^2\alpha, \quad \ldots\ldots(8)$$

where Y in the former case is the intersection distance from the line of centres of the projected vignetting stop and the pupil, in the latter case of the two vignetting stop projections; and hence the (A, α) curve is smooth at the point of transition shown in figure 3 *b*, with a slope at the common point of

$$\frac{dA}{d\alpha} = -\frac{2D}{\pi}\sec^2\alpha;$$

$\dfrac{d^2A}{d\alpha^2}$ at this point on the curve is discontinuous.

Typical curves are shown in figure 4. It is clear that these are further from straight lines than in the previous case, though the tangent at the point of transition lies close to the curve for much of its length, and the line joining the point where vignetting begins $\left(A=1, \alpha=\tan^{-1}\dfrac{2(R-1)}{D}\right)$ to the point where vignetting is complete $\left(A=0,\ \alpha=\tan^{-1}\dfrac{2R}{D}\right)$ gives a rough overall approximation.

If the pupil does not lie centrally between the vignetting stops, the oblique aperture will be reduced where the smaller angle equation applies, though this reduction will be negligible for small displacements, since the differential is zero in the symmetrical position. On the lower part of the curve, the aperture is unchanged over a finite range of relative movement.

In the most general case of two vignetting stops of different radii R_1, R_2 the problem is more complicated, since the position of the pupil between the stops which gives maximum aperture is that which equalizes the incidence

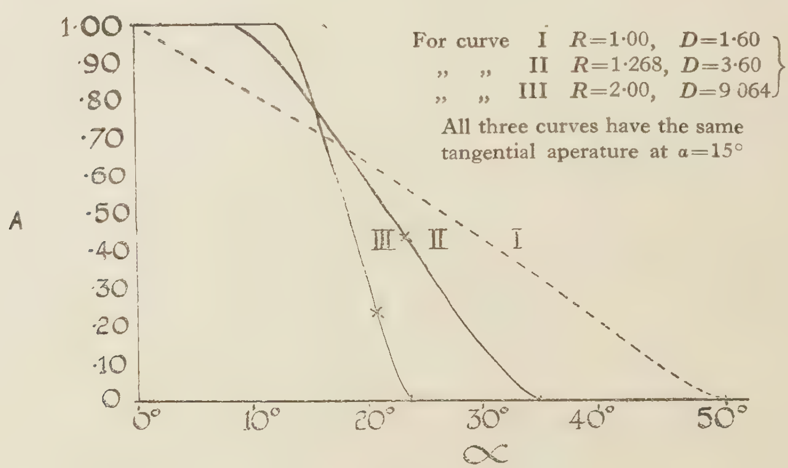

Figure 4.

heights of the projected stops with the pupil circle from the line of centres, and this condition gives different best relative positions at different angles. For small angles of field and degrees of vignetting, the distances of the pupil from the stops should be such that $D_1/D_2 = (R_1-1)/(R_2-1)$.

Either type of vignetting has a negligible effect in reducing the radial aperture except at very high degrees of vignetting, and this, of course, is what is required, since (as is implicit in a paper by H. H. Hopkins which has been submitted for the Optical Group) it is primarily in the tangential plane that the higher-order comatic aberrations are effective. In practice it has usually been found, when the necessary degree of vignetting to give reasonable definition has been introduced for some intermediate angle (say 15° to 18° for a lens covering 26° semi-angular field), that vignetting increases with increasing angles more rapidly than the aberrations require, giving the well known effect of improved definition at the edge of the field compared with the intermediate field zone. (This is often partly due to field-zonal astigmation, but in wide-aperture lenses of fairly wide angle is also due to this stopping down of comatic pencils.) Clearly if a tangential relative aperture A at angle α is required to give tolerable performance at this angle, this may be achieved either by vignetting stops of the same size as the entrance pupil relatively near to it, or by larger stops further from it; and since the former will give a smaller $dA/d\alpha$, with more vignetting at smaller angles, but less at larger ones, this will normally be the better form to adopt, since it will give a more uniform illumination over the whole field. As the angle of pencil changes, different stops within the lens may become effective as vignetting stops; but in this case those nearest to the pupil in size will become effective first, and the larger ones further from the pupil at larger angles, i.e., we shall pass with increasing angle along an intermediate section of curve from a curve of the type of figure 2 to one of the type of figure 4, with consequent more rapid vignetting at larger angles. In the lenses I have been working with, this is wholly undesirable, but in particular cases it may be required and may be very readily provided.

In general, it is clear that once a degree of lens correction has been achieved which allows some relaxation of the maximum vignetting, then it is both possible and desirable to choose vignetting stop positions and sizes to match the particular lens correction. Further, the sizes and positions of the vignetting stop images in the object space, relative to the entrance pupil, afford data from which the form of vignetting of a lens at any angle can readily be deduced, and such information might usefully be made available by manufacturers.

THE COMPUTATION OF ELECTRON TRAJECTORIES IN AXIALLY SYMMETRIC FIELDS

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Communicated by T. Smith, F.R.S.; MS. received 25 *August* 1944; *read* 27 *October* 1944

ABSTRACT. Existing methods for calculating electron paths are discussed and some factors limiting their accuracy and general application are pointed out. A new method, which overcomes these difficulties, is proposed. The equations of motion are integrated numerically in a most direct manner, and at each stage the accuracy may be checked by using the energy equation. The method may be used in a combined electric and magnetic field, and it is not limited to paraxial rays or to rays which are initially parallel to the axis. The necessary computations may be carried out rapidly with the aid of a computing machine.

§ 1. INTRODUCTION

In many problems in applied physics (and, in particular, in electron optics) the determination of electron trajectories is of considerable importance. In some cases the field is purely electrostatic, in others a magnetic field is also present; but in any case it is generally of such a nature that the trajectory cannot be obtained by direct integration of the equations of motion. In what follows we shall assume the field to be known (usually only numerically), but it is worth pointing out that methods exist for the calculation of the field itself. Shortley and Weller (1938) have given a method for the numerical solution of Laplace's equation with given boundary values, and this method has been improved by Frocht and Leven (1941). Southwell and Vaisey (1943) have recently applied relaxation methods to plane potential problems of a quite general nature.

A number of methods for calculating electron paths are based on the paraxial equation, which is of the form

$$2\Phi(z)\frac{d^2r}{dz^2}+\Phi'(z)\frac{dr}{dz}+\tfrac{1}{2}\Phi''(z)r=0 \qquad \ldots\ldots(1)$$

in the case of electrostatic fields ($\Phi(z)$ being the potential along the axis) and of the form

$$\frac{d^2r}{dz^2}+f(z)r=0 \qquad \ldots\ldots(2)$$

in the case of magnetic fields. The equation (2) and its related literature are discussed in the following paper (Goddard and Klemperer, 1944), but it is convenient here to give a short survey of the development of methods for solving the first equation. On writing $r(z)=P(z)/\{\Phi(z)\}^{1/4}$ the equation (1) reduces to the form (see Picht, 1939, p. 110)

$$P''(z)+\frac{3}{16}\left(\frac{\Phi'(z)}{\Phi(z)}\right)^2P(z)=0. \qquad \ldots\ldots(3)$$

By applying Picard's iterative method, Picht has given a solution of (3) in the form

$$P(z)=\sum_{\nu=0}^{\infty} P_\nu(z),$$

where

$$P_\nu(z)=-\frac{3}{16}\int_{z_0}^{z}\int_{z_0}^{\zeta_1}\left(\frac{\Phi'(\zeta)}{\Phi(\zeta)}\right)^2 P_{\nu-1}(\zeta)\,d\zeta\,d\zeta_1,$$

and thereby obtained formulae for the optical constants relating to an electron lens with field distribution $\Phi(z)$. Another transformation, obtained by writing

$$P(z)=\exp\left(\int^{z}\kappa(\zeta)\,d\zeta\right),$$

reduces (3) to the Riccati equation,

$$\frac{d\kappa}{dz}+\kappa^2+\{h(z)\}^2=0,$$

where

$$h(z)=\frac{\sqrt{3}}{4}\cdot\frac{\Phi'(z)}{\Phi(z)}.$$

This may be solved in closed form when $h(z)$ satisfies certain conditions, but in any case the solution may be obtained numerically. This has been done by Gans (1937), using the Runge-Kutta method. A method which is more suitable when $\Phi(z)$ is given experimentally is the Euler polygon method, applied by Gans to electron-optical problems. Still another approach is the parabola method of Recknagel (1937). In each of these cases explicit formulae for the focal length and other lens constants can be given in terms of $\Phi(z)$.

On the numerical side, other methods, less analytic than those mentioned above, have been developed for the use of computers; these include the method of joined circular segments and the action function method (Spangenberg and Field, 1942) and the method of Maloff and Epstein (1938). There are also methods of a graphical nature (e.g. Jacob, 1938), and allied with these is trigonometric ray-tracing (Klemperer and Wright, 1939), which is based on known procedures in geometrical optics. These methods have a restricted application (for example, according to Jacob errors as large as 50 per cent may be introduced in the method he describes), and, moreover, there is no convenient way of estimating or controlling the accuracy.

From the above survey it is evident that the whole subject of determining electron paths has been much in need of synthesis for a considerable time. Spangenberg and Field (1942), after consideration of existing methods, come to the conclusion that "all calculating methods are sufficiently long in application and indeterminate in accuracy that experimental methods of finding lens characteristics are preferred". It is this erroneous notion which we wish to dispel. In brief, the purpose of the present paper is to point out that once an electromagnetic field is given (either experimentally or by one of the computational methods mentioned above) the paths of electrons passing through this field may be found by a *computational* method which is rapid and general in its application and in which we have direct control over the accuracy.

§2. ADVANTAGES OF THE NEW METHOD

All the numerical methods mentioned above have one common feature which limits their accuracy and general application: if the electron is initially at the point P_0 and the points determined by the step-by-step method are $P_1, P_2, \ldots, P_n$, then the process by which we obtain the next point, P_{n+1}, is one which only utilizes a knowledge of the conditions (of electron and field) existing at the point P_n. Such methods are clearly inaccurate when, in the neighbourhood of P_n, the conditions of the field * or of the electron † are changing rapidly. In contrast to this, in the proposed method the passage from P_n to P_{n+1} involves the consideration of a considerable part of the past history of the electron, namely, the conditions existing not only at P_n but also at P_{n-1}, P_{n-2}, P_{n-3}, and, in cases where great accuracy is required, at P_{n-4} and P_{n-5}. It is applicable to cases where electric and magnetic fields co-exist and it is not limited, as are the above-mentioned methods, to paraxial rays nor to rays that are initially parallel to the axis. From the outset our method is radically different from previous methods. We commence, not with the paraxial equation, which *ipso facto* imposes a considerable restriction, but with the general equations of motion of the electron, and give formulae (taken for the most part from the calculus of finite differences) by which these may be solved numerically to give the path. ‡ This approach may be profitably compared with the method used by Hartree (1932/33) for solving differential equations of the type $d^2y/dx^2=f(x,y)$ which arise in the approximate calculation of atomic structures by the method of the self-consistent field. Two of the chief advantages of the method over existing methods are (i) rays which are initially skew or of large aperture may be traced (so yielding a method for studying errors of electrostatic or magnetic lenses); (ii) there is direct control over the accuracy, which may be estimated by checking the energy equation, and this control makes the method one of more general application than some of the methods mentioned above, which have been mostly designed to treat particular types of problems.

§3. THE EQUATIONS OF MOTION

In an electromagnetic field, defined by an electric scalar-potential function ϕ and a magnetic vector-potential function A, the equation of motion of an electron is

$$\mathbf{F}=m\mathbf{a}=\frac{e}{c}\,\mathbf{u}\times\mathbf{H}-e\,\mathrm{grad}\,\phi,$$

where $\mathbf{H}(=\mathrm{curl}\,\mathbf{A})$, $\mathbf{F}$, $\mathbf{a}$ and $\mathbf{u}$ are the magnetic field strength, the force on the electron and the acceleration and velocity of the electron respectively. In any orthogonal co-ordinate system, set up in the region covered by the field, this

* For example, in an electrostatic field, the method of Klemperer and Wright involves the assumption that the spaces between the equipotentials V_1 and V_2, V_2 and $V_3, \ldots$ are of constant refractive indices $\mu_i=\sqrt{V_i}$ $(i=1, 2, 3, \ldots)$. Such an assumption will clearly be inaccurate at places where $\partial V/\partial s$ (taken along the path) has a high value, unless the number of subdivisions is increased, in which case the amount of necessary computation may become excessive.

† For example, if $\partial v/\partial s$ (where v is the velocity of the electron) has a high value, as happens along a portion of the path in the case of the reflection of an electron by an electron mirror.

‡ The method is entirely numerical, there being no graphical considerations at any stage, and no appeal to tables of functions. As a result, the work may be carried out very rapidly with a high-speed electrical calculator.

vector equation may be resolved into three simultaneous scalar equations, which yield, upon integration, the path of the electron. We consider only axially symmetric fields, because in practice most fields are of this type. In such cases, using a cylindrical co-ordinate system (r, z, ψ), the equations of motion reduce to the form (see Picht, 1939, p. 17)

$$\left.\begin{aligned} m\ddot{z} &= -\frac{\partial M}{\partial z}, \\ m\ddot{r} &= -\frac{\partial M}{\partial r}, \\ r^2\dot{\psi} &= C - \frac{e}{m} rA, \end{aligned}\right\} \qquad \ldots\ldots(4)$$

where $A(=A_\psi)$ is the only non-vanishing component of $\mathbf{A}$,

$$M = \frac{m}{2}\left(\frac{C}{r} - \frac{e}{m}A\right)^2 + e\phi = \frac{m}{2} r^2\dot{\psi}^2 + e\phi$$

and

$$C = r_0{}^2\dot{\psi}_0 + \frac{e}{m} r_0 A_0,$$

$r_0, \dot{\psi}_0$ and A_0 being the initial values. It is to be particularly noticed that, since M is independent of ψ, the motion in an axial plane is independent of the motion in an equatorial plane. The angular motion, however, enables us to deduce an important result concerning the electron path. Since the vector potential $\mathbf{A}$ vanishes on the axis, it follows from (4.3) that for small values of r, $\dot{\psi} = O(1/r^2)$, provided $C \neq 0$. This means that as the electron approaches the axis the angular velocity rapidly increases. An infinite value for such a velocity is not physically admissible, so that the electron can never meet the axis. Now the constant C cannot be zero in the case of initially skew rays $(\dot{\psi}_0 \neq 0)$, so that we have the result: *Rays which are initially skew to the axis cannot intersect the axis.*

There are two important special cases of equations (4):

I. *Purely electric field* $(A \equiv 0)$. The equations reduce to

$$\left.\begin{aligned} m\ddot{z} &= -e\frac{\partial \phi}{\partial z}, \\ m\ddot{r} &= -e\frac{\partial \phi}{\partial r}, \\ r^2\dot{\psi} &= r_0{}^2\dot{\psi}_0. \end{aligned}\right\} \qquad \ldots\ldots(5)$$

II. *Purely magnetic fields* $(\phi \equiv 0)$. We consider cases in which the electron is initially at a point where the field strength is negligible $(A_0 = 0)$.

(i) *Path initially skew to the axis* $(\dot{\psi}_0 \neq 0)$. The equations (4) become

$$\left.\begin{aligned} m\ddot{z} &= e\left(\frac{C}{r} - \frac{e}{m}A\right)\frac{\partial A}{\partial z}, \\ m\ddot{r} &= e\left(\frac{C}{r} - \frac{e}{m}A\right)\left[\frac{1}{r}\frac{\partial}{\partial r}(rA) + \frac{m}{er}\left(\frac{C}{r} - \frac{e}{m}A\right)\right], \\ r^2\dot{\psi} &= C - \frac{e}{m} rA. \end{aligned}\right\} \qquad \ldots\ldots(6)$$

(ii) *Path initially in an axial plane* ($\dot{\psi}_0 = 0$). We now have $C=0$, and the equations take the particularly simple form

$$\left.\begin{aligned} \ddot{z} &= -\left(\frac{e}{m}\right)^2 A \frac{\partial A}{\partial z}, \\ \ddot{r} &= -\left(\frac{e}{m}\right)^2 A \frac{\partial A}{\partial r}, \\ \dot{\psi} &= -\left(\frac{e}{m}\right) \frac{A}{r}. \end{aligned}\right\} \qquad \ldots\ldots(7)$$

These last equations are used in the following paper (Goddard and Klemperer, 1944) to trace rays through magnetic lenses. But it is appropriate here to point out that apart from this treatment the only other valid approach is one based on the paraxial equation (2). It is not possible, for example, to adapt for magnetic lenses any of the above-mentioned methods which are based on equation (1). This is because of the fact that, in the case of electrostatic lenses, the potential $\phi(r, z)$ satisfies Laplace's equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

and the derivation of (1) uses this fact; whereas in the case of magnetic lenses the vector potential $\mathbf{A} = (0, 0, A)$ satisfies

$$-(\text{curl curl } \mathbf{A})_\psi \equiv \frac{\partial H_z}{\partial \rho} - \frac{\partial H_\rho}{\partial z} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - \frac{A}{r^2} + \frac{\partial^2 A}{\partial r^2} = 0,$$

so that A^2, which is effectively the potential in an axial plane in equation (7) cannot satisfy Laplace's equation.

From an examination of equations (4), (5), (6) and (7) it will be seen that the motion in an axial plane is, in all cases, governed by equations of the form

$$\ddot{z} = f(z, r)$$
$$\ddot{r} = g(z, r).$$

These equations do not involve the first-order derivatives $\dot{z}$ or $\dot{r}$, and for purposes of numerical integration several special methods have been devised which take advantage of this fact. A comprehensive discussion of these is given in the monograph by Bennett, Milne and Bateman (1933). There are two types: (1) using differences, and (2) using ordinates. Both have been used, but the latter was found to involve considerably less computation. We give a short description of a method which has proved very satisfactory. It is most direct compared with previous methods, because it is unnecessary to calculate any auxiliary quantities. A detailed application is made in the following paper.

§ 4. PRACTICAL USE OF THE METHOD

Suppose the initial conditions of the problem are $z = z_0$, $r = r_0$, $\dot{z} = \dot{z}_0$, $\dot{r} = \dot{r}_0$. The numerical work is arranged in tabular form as below:

t	$\ddot{z}$	$\ddot{r}$	$\dot{z}$	$\dot{r}$	z	r	
t_0	$\ddot{z}_0$	$\ddot{r}_0$	$\dot{z}_0$	$\dot{r}_0$	z_0	r_0	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	A
.	.	.	.	.	.	.	
t_s	z_s	$\ddot{r}_s$	$\dot{z}_s$	$\dot{r}_s$	z_s	r_s	
τ_{s+1}	$\ddot{z}_{s+1}$	$\ddot{r}_{s+1}$	$\dot{z}_{s+1}$	$\dot{r}_{s+1}$	z_{s+1}	r_{s+1}	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	B
τ_n	$\ddot{z}_n$	$\ddot{r}_n$	$\dot{z}_n$	$\dot{r}_n$	z_n	r_n	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	

Section A, which provides a set of starting values for section B, is obtained by assuming the path to be locally parabolic.* This is certainly accurate if the electron is initially at a point where the field strength is weak, and, in any case, can be made as accurate as is desired by choosing a sufficiently small time interval $w = t_{i+1} - t_i$. The laws of uniform motion provide us with the formulae

$$\dot{z}_{m+1} \approx \dot{z}_m + w\ddot{z}_m,$$
$$z_{m+1} \approx z_m + w\dot{z}_m + \tfrac{1}{2}w^2\ddot{z}_m$$

for z and similar formulae for r. An iteration based on these enables section A to be built up step by step. Section B is obtained by using more accurate formulae due to W. E. Milne (1933).† These are

$$z_{n+1} \approx z_n + z_{n-2} - z_{n-3} + \frac{\omega^2}{4}(5\ddot{z}_n + 2\ddot{z}_{n-1} + 5\ddot{z}_{n-2}),$$

which is exact if fourth differences of z are negligible, and

$$z_{n+1} \approx z_n + z_{n-4} - z_{n-5} + \frac{\omega^2}{48}(67\ddot{z}_n - 8\ddot{z}_{n-1} + 122\ddot{z}_{n-2} - 8\ddot{z}_{n-3} + 67\ddot{z}_{n-4}),$$

which is exact if sixth differences of z are negligible.‡ Here $\omega = \tau_{i+1} - \tau_i$, and similar formulae hold for r.

Regarding the paraxial equations (1) and (2), it will be noticed that the method of integration just described is directly applicable to these cases, since the first derivative dr/dz is absent in (2), and (1) can be transformed into (3), in which, again, the first derivative is absent. This method of numerical integration is easily the best in these cases. It is superior to the numerical methods described in § 1, because it is much quicker and more accurate. However, the approach starting with the general equations is better still, because here we

* In cases where the electron is initially at a point where the field strength is negligible ($\ddot{z}_0 = \ddot{r}_0 = 0$) the path may be assumed linear for the construction of section A. Then it is possible to dispense with the columns $\dot{z}$ and $\dot{r}$. These are the conditions existing in the following paper.

† Note that the time interval $\omega = \tau_{i+1} - \tau_i$ may be different from w. It must be determined by considerations of accuracy. The use of the more accurate formulae would often allow us to take $\omega > w$.

‡ These formulae are very convenient for use in conjunction with an electrical calculator. The right-hand side may be evaluated *en bloc* on the machine, that is, without writing down any interim calculations.

have a ready means of estimating and controlling the accuracy. It is evident from the above table that the columns relating to $\ddot{z}$ and $\ddot{r}$ may be quickly integrated numerically to give z and $\dot{r}$ at any desired stage. Then $\dot{\psi}$ may be computed directly from (5.3), (6.3) or (7.3). With these three quantities calculated, the energy equation may be checked, and in this way an estimate may be obtained of the accuracy. Regarding errors arising from the use of approximations, we have a ready means of controlling these, because a difference table of the values of z and r at points of the path reveals whether or not we have used a sufficiently accurate approximation for z_{n+1} and r_{n+1} in terms of previous ordinates.

ACKNOWLEDGMENTS

Finally, I wish to thank Mr. I. Shoenberg, the Director of Research at Electric and Musical Industries, Ltd., and also Mr. G. E. Condliffe, for permission to submit the above results for publication.

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ELECTRON RAY TRACING THROUGH MAGNETIC LENSES

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Communicated by T. Smith, F.R.S.; MS. received 25 *August* 1944; *read* 27 *October* 1944

ABSTRACT. A new numerical method is presented for computing electron paths through magnetic lenses. The process is direct and accurate and is not confined to paraxial rays, nor necessarily to rays that are initially non-skew. There is direct control over the error, since the energy equation may be checked at any stage. The method agrees well with experiment and has been used to determine the spherical aberration curve.

§1. INTRODUCTION AND SURVEY OF PREVIOUS WORK

In view of their importance in electron optics and their application to β-ray spectrometry, magnetic lenses have recently been the subject of study by several authors (Klemperer, 1935; Davies and O'Ceallaigh, 1937; Siegbahn, 1942). For every electron-optical investigation, an exact knowledge

of the paths of the electrons is very desirable, but so far there have not been presented really satisfactory methods of ray tracing through magnetic lenses. It has not been possible, given the coil and pole pieces, to compute the magnetic field distribution and, even when the field is supplied by experimental data, the equations of motion are not of a form which readily lends itself to integration. For this reason attention has been confined so far only to the treatment of paraxial rays,* for which one can make considerable progress, starting with the equation

$$\frac{d^2r}{dz^2}+\frac{er}{8mU}H^2(z)=0, \qquad \ldots\ldots(1)$$

where r and z belong to a cylindrical co-ordinate system, $H(z)$ is the magnetic field strength along the axis, and U is the energy in electron volts. We give a short account of recent work that has taken place along these lines.

A circular current of radius a is known to have the field distribution $H(z)=H_0\Big/\left[1+\left(\frac{z}{a}\right)^2\right]^{3/2}$, and this case has been studied in detail by Wallauschek and Bergman (1935). This form of $H(z)$ has led Glaser (1941) to approximate to the field of a magnetic lens by means of the formula $H(z)=H_0\Big/\left[1+\left(\frac{z}{a}\right)^2\right]^{\mu}$, where $a=\dfrac{d}{\sqrt{\{\sqrt[\mu]{2}-1\}}}$ (d being the half-value width of the distribution curve of $H(z)$), while μ is obtained by a process of curve fitting. This allows equation (1) to be transformed into one of Hill's type, namely (using Glaser's notation),

$$v''(\phi)+J(\phi)v(\phi)=0, \qquad \ldots\ldots(2)$$

where $J(\phi)=1+\kappa^2\sin^{4(\mu-1)}\phi$. Glaser investigates in detail the case $\mu=1$, when this equation can be exactly integrated in terms of circular functions. It is possible to go a considerable distance in the study of a lens having such a field distribution. In particular, formulae are obtained for the focal length (f), the mid-focal length (z_f), the spherical aberration and the chromatic aberration. Dosse (1941), using these formulae, has made some computations in the case of lenses having a non-symmetrical field form, that is,

$$H(z)=H_0\Big/\left[1+\left(\frac{z}{a}\right)^2\right]\ (z>0) \quad \text{and} \quad H(z)=H_0\Big/\left[1+\left(\frac{z}{b}\right)^2\right]\ (z<0).$$

The case where μ is any integer ($\geqslant 1$) has been investigated by Svartholm (1942). The limiting case as $\mu\to\infty$ has only recently been treated by K. Siegbahn (1944). There the field distribution approaches the form $H(z)=H_0\,.\,e^{-(z/b)^2}$, where $b=\dfrac{d}{\sqrt{\log 2}}$.† A series solution of (1) is given when $H(z)$ is of this

* An exception to this is the series of papers by Grünberg (1942 and 1943). Here a study is made of the conditions necessary on an electromagnetic field and a beam of ions in order that a focusing action may take place. The treatment is very general and hardly relates to the subject of the present paper.

† Siegbahn does not make this clear. If the limiting form is $H(z)=H_0/L$,

then $$L=\lim_{\mu\to\infty}\left[1+\left(\frac{z}{a}\right)^2\right]^{\mu}=\lim_{\mu\to\infty}\left[1+\frac{1}{\mu}\left(\frac{z}{d}\right)^2\mu\,(2^{1/\mu}-1)\right]^{\mu}=e^{(z/b)^2},$$

where $$b=d\Big/\sqrt{\left\{\lim_{\mu\to\infty}\mu\,(2^{1/\mu}-1)\right\}}=d/\sqrt{\log 2},$$

since $$\lim_{\mu\to\infty}\left(1+\frac{\log 2}{\mu}\right)^{\mu}=2.$$

form, and corrections are obtained to the formulae of Glaser for f and z_f. It is shown that these corrected formulae are in much closer agreement with experiment.

It will be seen from the above survey that in the study of magnetic lenses the tendency has been to give analytical treatments based on the equation of motion for a paraxial electron. There are two obvious limitations in adopting such an approach: firstly, beams of large aperture cannot be treated, and secondly, no provision can be made for the study of paths which are initially *skew*. The latter is a particularly serious drawback considering the importance of skew rays in the study of image errors.

Regarding graphical and numerical methods, it has been pointed out in the previous paper (Goddard, 1944) that several of the existing methods for ray-tracing are applicable to electrostatic but not to magnetic electron lenses. A process which can, in principle, be used in both cases is the orthodox method of glass optics (Klemperer and Wright, 1939), in which rays are traced by considering the refraction through the equipotentials according to Snell's law. For this purpose a knowledge of the meridional magnetic potential distribution is required, and this is obtained from an experimentally determined magnetic field distribution in a manner described by Dosse (1936) and Klemperer (1939). However, the results of such ray-tracing are unsatisfactory for the reason that the rays cut the equipotentials at small angles,* especially near the centre of the lens. Here the refractive power of the lens is greatest, and it is just in this neighbourhood that the ray-tracing method is most inaccurate. Siday (1942), starting with the paraxial equation (1), has described a process which gives the focal length and the positions of the principal planes. Several approximations are made at various stages, but there is no discussion to show that the integrated effect of these produces only a negligible error.† Moreover, the paraxial equation (1) is derived on the assumption that $(dr/dz)^2$ is negligible, that is, $(dt/dz)^2 \ll 1$ (see Glaser, 1933). However, in a treatment where the values of r are derived from a knowledge of those of dr/dz, it would seem necessary to commence with an equation not so specialized as (1). Apart from this, the method is subject to the same limitations as the analytical treatments mentioned above, namely, the exclusion of an application to skew rays and to beams of large aperture.

In the present paper we overcome these limitations by adopting a new approach, based on a numerical method devised by one of us (Goddard, 1944). The equations of electron motion (not confined to the paraxial case) are integrated numerically, and we thus have a direct determination of the path. After a short discussion of the experimental determination of the field distribution of the

* Brüche (1933) has introduced the terms "transgradient" and "congradient" for fields which are traversed by electrons substantially in the direction of or normal to the equipotentials. For the former type the orthodox methods of ray tracing are impracticable.

† Some of the mathematical steps appear to require more justification than the author has given. For example, an integration of (1) by parts yields

$$\left(\frac{dr}{dz}\right)_2 - \left(\frac{dr}{dz}\right)_1 + \chi\left\{r_2\int^2 H_z^2 dz - r_1\int^1 H_z^2 dz\right\} - \chi\int_1^2\left(\frac{dr}{dz}\int H_z^2\,dz\right)dz = 0,$$

where we have written $\chi = e/8mU$, and it is not clear that the last integral is negligible, as Siday assumes.

lens, the computational method is described. It is shown how to integrate (numerically) the equations of motion once the values A, $A\,\frac{\partial A}{\partial z}$ and $A\,\frac{\partial A}{\partial r}$ are found from the experimental field distribution (A being the magnetic vector potential). A simple and rapid method based on the paraxial equation is also given for the sake of completeness. An experimental method of ray tracing, using a sliding fluorescent target, is then briefly described. The results obtained by the two computational methods and by the experimental method are compared and the accuracy is discussed.

§2. EXPERIMENTAL FIELD PLOTTING

A preliminary to ray tracing is the experimental determination of the axial component of the magnetic field. The application of the usual search-coil method to the measurement of the field strength of magnetic lenses has been described by Klemperer (1935). Small search coils of only a few millimetres diameter have been recommended to obtain a high accuracy and to measure small variations in the field. To obtain sufficient sensitivity the search-coil

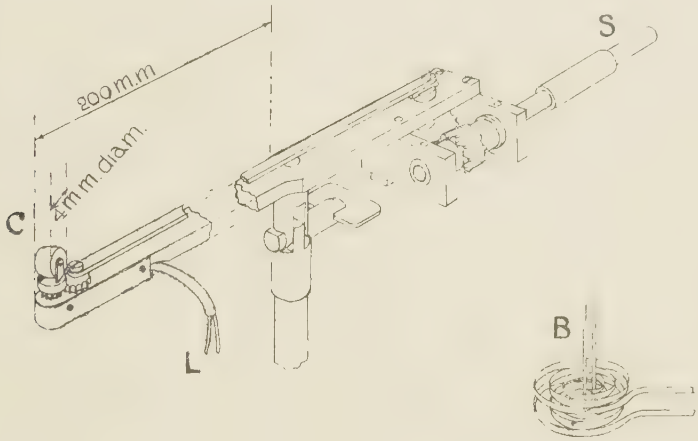

Figure 1. Search-coil oscillator.

oscillator described by Klemperer and Miller (1939) has been used, the output energy being increased by a resonance valve amplifier. Some constructional simplifications and improvements have, however, been introduced recently, in particular, the oscillating slide, which was incorporated in the first model, has been eliminated. The new model is shown in figure 1. There, as in the old arrangement, S is a shaft coupled to a synchronous motor, L are the leads to a 10-c.p.s. resonance amplifier, and B (not shown in the main figure) are two spirals connecting the oscillating coil C to the fixed leads L. The search coil itself has inner and outer radii of 1 mm. and 2 mm. respectively, and now consists of 500 turns of wire of diameter 0·05 mm.

In an alternative method, developed by Sandor (1941), large search coils may be used if a series of them with various diameters is arranged to be strictly coaxial with the lens. If these coils are brought into position one by one and then rapidly removed from the lens along its axis, the integral flux in the axial direction

is measured. One important difference between the two methods relates to resonance amplification. This is desirable for the elimination of spurious currents, and it involves the use of rotating or oscillating search coils. It may be used in conjunction with Sandor's method in the case of the smaller coils, but to measure the marginal lens field the coil has to be nearly as large as the inner diameter of the pole pieces, and there are clearly mechanical difficulties in the rotation of the coil. The method of Klemperer and Miller is preferable in this case. It is also to be noticed that, in the last-mentioned method, only one search coil is necessary, whereas in Sandor's method a large number of coils of different diameters is needed to investigate lenses of various diameters.

§ 3. COMPUTATIONAL METHOD

The equations of motion of an electron in a magnetic field of axial symmetry may be written in the form (see preceding paper)

$$\left.\begin{aligned} \ddot{z} &= -\left(\frac{e}{m}\right)^2 A\frac{\partial A}{\partial z}, \\ \ddot{r} &= -\left(\frac{e}{m}\right)^2 A\frac{\partial A}{dr}, \\ \dot{\psi} &= -\left(\frac{e}{m}\right)\frac{A}{r}, \end{aligned}\right\} \qquad \ldots\ldots(3)$$

where A is the magnetic vector potential and r, z and ψ are the co-ordinates of a cylindrical system. The electron is assumed to move initially in an axial plane ($\dot{\psi}_0 = 0$). Before integrating these equations we calculate A, $\frac{\partial A}{\partial z}$ and $\frac{\partial A}{dr}$.

The vector potential $A = A(r, z)$, given by

$$A(r, z) = \frac{1}{r}\int_0^r \rho H(\rho, z)\, d\rho,$$

is obtained by numerical integration, the values of $H(\rho, z)$ being obtained experimentally. For this purpose a sufficiently accurate formula is

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\{f(a)+f(b)\} + \Delta x \sum_{i=1}^{n-1} f(x_i),$$

where the interval (a, b) is divided into n intervals (x_i, x_{i+1}) each of length Δx. From this formula we obtain

$$A(r, z) \approx [\tfrac{1}{2}H(r, z) + \frac{1}{r}\sum_i r_i H(r_i, z)]\Delta r.$$

The values of A are calculated at the points of a lattice L obtained by taking lines parallel to the z and r axes in an axial plane. Then the value of A at any point within one of the squares of the lattice is computed by linear interpolation. To achieve a high order of accuracy, a finer mesh is used in the central part of the lens.

Having constructed the lattice L, the derivatives $\partial A/\partial r$ and $\partial A/\partial z$ at a point of L may be computed by numerical differentiation. However, such a process involves additional approximations and, if possible, it is desirable to have an

exact method. We give such a method for computing $\partial A/\partial r$, and this is valuable for the additional reason that the lattice L is not very convenient for the computation of $\partial A/\partial r$ by numerical differentiation. The values of $\partial A/\partial z$ must,

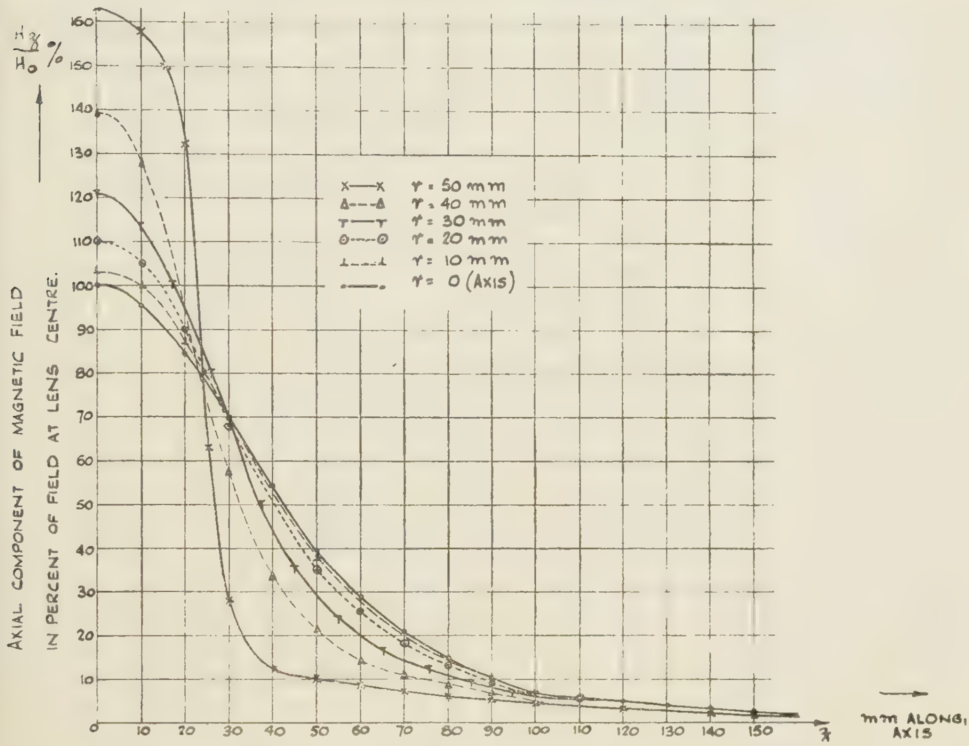

Figure 2. Field distribution in magnetic lens.

however, be obtained by a method involving differences. The expression for A may be written in the form

$$A(r, z) = \frac{1}{r} I(r, z),$$

where

$$I(r, z) = \int_0^r \rho H(\rho, z) \, d\rho.$$

It follows that

$$\frac{\partial A}{\partial r} = \frac{r \dfrac{\partial I(r, z)}{\partial r} - I(r, z)}{r^2} = H(r, z) - \frac{1}{r} A(r, z).$$

The derivative $\partial A/\partial z$ may be computed at the point $P_0(r_0, z_0)$ (where $A = A_0$) by neglecting differences of the function $A(r_0, z_0)$ beyond the second.† Let ω be the interval of the lattice L and suppose the values of A at the points $(r_0, z_0 \pm \omega)$ and $(r_0, z_0 \pm 2\omega)$ are A_1, A_1^* and A_2, A_2^* respectively. Then for $\partial A/\partial z$ we obtain

† An alternative method is to write $\frac{\partial A}{\partial z} = \frac{1}{r}\int_0^r \rho \frac{\partial H(\rho, z)}{\partial z} d\rho$ and compute $\partial A/\partial z$ by numerical integration of $\rho \frac{\partial H}{\partial z}$. The computation of $\frac{\partial H(\rho, z)}{\partial z}$ would involve the numerical differentiation of the initial family of curves in figure 2, so that this method has no advantage over the one given.

the two expressions $2A_1 - \frac{3}{2}A_0 - \frac{1}{2}A_2$ and $\frac{3}{2}A_0 - 2A_1^* + \frac{1}{2}A_2^*$, so that, taking the mean, we have

$$\omega\frac{\partial A}{\partial z} \approx (A_1 - A_1^*) - \tfrac{1}{4}(A_2 - A_2^*).$$

By means of these formulae, the values of $\partial A/\partial r$ and $\partial A/\partial z$ are calculated at the points of the lattice L. The new lattices L_1 and L_2 are constructed, these being of values of $A\dfrac{\partial A}{\partial z}$ and $A\dfrac{\partial A}{\partial r}$. With the aid of these, equations (3) are integrated simultaneously, using a step-by-step method described in the previous paper.

In order to illustrate the method, the details are now given of an example which refers to the symmetrical magnetic lens having the field distribution shown in figure 2. The lattices L, L_1, L_2 for this case are shown in tables

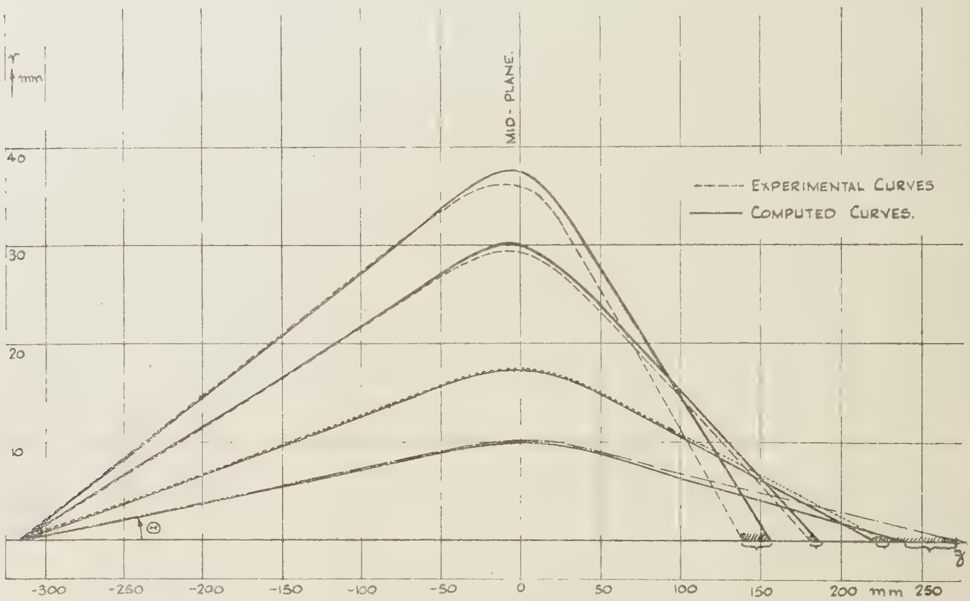

Figure 3. Tracing of initially homocentric bundle of 500-volt electrons through the meridional plane. $H_0 = 16{\cdot}5$ oersteds.

1, 2 and 3. We have taken a mesh length of 0·5 cm. from the centre to the point $z = 4$ cm. and a mesh length of 1·0 cm. from $z = 4$ cm. to $z = 9$ cm., beyond which point the field strength is practically negligible. This allows us to use linear interpolation in finding the value of the function at an internal point.† It should be noted that in order to obtain values which can be utilized for any coil current within a certain range of the given lens arrangement, the tables give the values of σA, $\frac{1}{2}\sigma^2 A\dfrac{\partial A}{\partial z}$ and $\frac{1}{2}\sigma^2 A\dfrac{\partial A}{\partial r}$, where $\sigma = 100/H_0 = 6{\cdot}06$,

† If the values of a scalar function ϕ at the vertices of the square $P_0 P_1 P_2 P_3$ are $\phi_0, \phi_1, \phi_2, \phi_3$, the value at an internal point P is given by $A\phi = \sum_{i=0}^{3} A_i\phi_i$, where A is the area of the square and A_0, A_1, A_2, A_3 are the areas of the rectangles into which the square is divided by lines drawn through P parallel to the sides (A_i is the area opposite P_i).

Tables relating to the field distribution of figure 2

Table 1. The lattice L $\left(\text{values of } \sigma A \text{ where } \sigma = \dfrac{100}{H_0}\right)$

r \ z	0·0	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	5·0	6·0	7·0	8·0	9·0
0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
0·5	25·0	25·0	24·0	23·0	21·5	19·5	17·5	15·5	13·5					
1·0	51·5	51·0	49·5	46·5	43·0	39·0	35·0	30·5	26·5	19·0	14·0	10·0	7·0	5·0
1·5	79·5	78·5	76·0	71·0	65·0	58·5	52·0	45·0	39·0					
2·0	109·0	107·5	103·5	96·5	87·5	78·5	69·0	59·5	51·0	36·5	26·5	19·0	13·5	9·5
2·5	141·0	138·0	133·0	123·0	111·5	99·0	86·5	73·0	61·5					
3·0	171·5	168·5	161·0	149·5	135·0	119·5	103·5	87·0	72·5	50·5	36·0	25·5	18·5	13·5
3·5	208·5	204·5	195·0	179·5	159·5	138·5	117·5	96·0	79·0					
4·0	243·5	238·5	227·0	208·0	183·5	158·0	132·5	107·0	87·0	59·0	42·0	30·0	22·5	16·0

Table 2. The lattice L_1 $\left(\text{values of } -\tfrac{1}{2}\sigma^2 A \dfrac{\partial A}{\partial z}\right)$

r \ z	0·0	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	5·0	6·0	7·0	8·0	9·0
0·0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0·5	0	13	25	35	40	41	35	31	27					
1·0	0	45	118	163	167	156	153	137	93	59	29	18	10	6
1·5	0	128	295	426	423	380	364	304	215					
2·0	0	296	582	844	820	726	681	558	376	219	108	59	30	20
2·5	0	587	1014	1445	1380	1238	1168	976	600					
3·0	0	969	1590	2056	2109	1897	1747	1436	888	429	207	105	49	35
3·5	0	1483	2486	3411	3449	2926	2629	1980	1057					
4·0	0	2117	3519	4862	4840	4069	3561	2595	1359	572	270	131	70	52

Table 3. The lattice L_2 $\left(\text{values of } \frac{1}{2}\sigma^2 A \frac{\partial A}{\partial r}\right)$

r \ z	0·0	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	5·0	6·0	7·0	8·0	9·0
0·0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0·5	644	625	588	518	452	380	306	233	176					
1·0	1326	1300	1225	1081	925	770	613	465	351	190	98	50	26	13
1·5	2147	2067	1951	1692	1452	1170	932	675	488					
2·0	3297	3185	2911	2521	2002	1580	1156	840	599	297	156	81	42	23
2·5	4061	3988	3611	3124	2615	2049	1531	1069	1027					
3·0	5388	5235	4728	4034	3308	2519	1837	1175	731	307	144	73	40	24
3·5	7134	7011	6268	5090	4062	2765	1905	1131	649					
4·0	9390	9132	8144	6344	4324	2726	1549	816	446	184	84	53	38	20

Table 4. Computation of an electron path using the general equations

$t\times10^9$	$\ddot{z}\times10^{-18}$	$\ddot{r}\times10^{-18}$	$\dot{z}\times10^{-9}$	z (cm.)	$\dot{r}\times10^{-9}$	r (cm.)	$\dot{\psi}$	ψ (rad.)	$\dot{r}^2+\dot{z}^2+r^2\dot{\psi}^2$
0	0·0000	0·0000	1·3260	−12·9780	0·1342	1·9254	0·0000	0·0000	1·776
1	0·0000	0·0000		−11·6520		2·0596	0·0060	0·0030	
2	0·0000	0·0000		−10·3260		2·1938	0·0086	0·0103	
3	−0·0004	−0·0004		−9·0000		2·3280	0·0135	0·0214	
4	−0·0009	−0·0009		−7·6745		2·4617	0·0210	0·0386	
5	−0·0023	−0·0021		−6·3498		2·5946	0·0323	0·0653	
6	−0·0062	−0·0051		−5·0276		2·7252	0·0493	0·1061	
7	−0·0183	−0·0170	1·3070	−3·7121	0·1172	2·8504	0·0785	0·1700	1·772
8	−0·0316	−0·0439		−2·4169		2·9560	0·1181	0·2683	
9	−0·0297	−0·0770		1·1491		3·0174	0·1527	0·4037	
10	0·0030	−0·0906		0·0907		3·0048	0·1656	0·5628	
11	0·0302	−0·0687		1·3361		2·9059	0·1480	0·7196	
12	0·0258	−0·0363		2·5975		2·7382	0·1124	0·8498	
13	0·0118	−0·0173		3·8921		2·5259	0·0743	0·9432	
14	0·0043	−0·0045		5·2069		2·3022	0·0484	1·0045	
15	0·0015	−0·0020	1·3125	6·5155	−0·2306	2·0789	0·0325	1·0450	1·780
16	0·0005	−0·0008	Δz 1·3068	7·8301	Δr −0·2277	1·8441	0·0210	1·0717	
17	0·0000	0·0000	1·3141	9·1369	−0·2406	1·6164	0·0132	1·0888	
18	0·0000	0·0000	1·3218	10·4510	−0·2317	1·3758	0·0087	1·1000	
19	0·0000	0·0000		11·7728		1·1441	0·0060	1·1104	

and not the direct values of A, $A\dfrac{\partial A}{\partial z}$ and $A\dfrac{\partial A}{\partial r}$. The details of the calculations relating to a particular path are shown in table 4. The electron is projected in a meridional plane (from a point on the axis at a distance of 32 cm. from the centre of the lens) at an angle θ to the axis where $\tan\theta = 0{\cdot}101$. The initial velocity components are $\dot{z}_0 = 1{\cdot}3260 \times 10^9$ cm. sec.$^{-1}$ and $\dot{r}_0 = 0{\cdot}1342 \times 10^9$ cm. sec.$^{-1}$. The time interval (ω) is 10^{-9} sec. (the time origin being arbitrary), and up to the line $t=3$ the path is linear, so that the values of z and r may be written down immediately. From the line $t=3$ to the line $t=7$, equations (3.1) and (3.2) are integrated by means of the formula (Goddard, 1944)

$$z_{n+1} \approx z_n + z_{n-2} - z_{n-3} + \frac{\omega^2}{4}\{5\ddot{z}_n + 2\ddot{z}_{n-1} + 5\ddot{z}_{n-2}\}, \qquad \ldots\ldots(4)$$

with a similar formula for r. On the $(n+1)$th line, the values of z_{n+1} and r_{n+1} are first calculated and then $\ddot{z}_{n+1}$ and $\ddot{r}_{n+1}$, using the lattices L_1 and L_2 and the original equations (3.1) and (3.2). On any line, the value of $\dot{\psi}$ given by (3.2) is derived, by using for r the value already obtained, and for A a value obtained from the lattice L. The values of ψ listed in the eighth column are given by numerical integration of the preceding column. Usually these values are not required, but we have included them here for completeness and also because in § 5 we make a comparison with experimental values. From the line $t=8$ onwards the formula for integration corresponding to (4) is

$$z_{n+1} \approx z_n + z_{n-4} - z_{n-5} + \frac{\omega^2}{48}\{67\ddot{z}_n - 8\ddot{z}_{n-1} + 122\ddot{z}_{n-2} - 8\ddot{z}_{n-3} + 67\ddot{z}_{n-4}\}. \qquad \ldots\ldots(5)$$

The process is stopped when the field strength is negligible, the path being once more linear. Differences Δz and Δr (shown at the bottom of columns 4 and 6) of consecutive entries of the z-column and the r-column respectively are now formed in order to derive mean values $\overline{\Delta z}$ and $\overline{\Delta r}$ for use in determining the point where the electron path cuts the axis. If this point is $(0, z)$ and if (r_1, z_1) is any point on the linear portion of the path, we have

$$z = z_1 + r_1\,\overline{\Delta z}/\overline{\Delta r}.$$

The values of Δz and Δr shown in table 4 are sufficient in order to see that it is unnecessary to carry the computations to four decimal places. The original curves of figure 1 (and hence the tables 1, 2 and 3) do not provide values of A, $A\dfrac{\partial A}{\partial z}$ and $A\dfrac{\partial A}{\partial r}$ with sufficient accuracy to ensure the fourth decimal place being correct. However, the computations have, in the above example, been carried to four places to ensure that negligible error is incurred in purely arithmetical processes.

Regarding the accuracy, it will be seen that in columns 4 and 6 we have listed (in three cases) values of $\dot{z}$ and $\dot{r}$ obtained by numerical integration of $\ddot{z}$ and $\ddot{r}$. These are used, together with the corresponding values of $\dot{\psi}$, to compute $\dot{r}^2 + \dot{z}^2 + r^2\dot{\psi}^2$ (shown in the 10th column), which should be constant, in view of the energy equation. We thus have a direct measure of the accuracy of the path when determined numerically. This measurement is rapidly obtained and is one of the special features of the method.

The paraxial equation (1) lends itself very readily to the method of integration described, and the computations may be carried out in a matter of minutes. The lattices L, L_1 and L_2 are unnecessary in this case, for if a dash (′) denotes differentiation with respect to z, the paraxial equation may be written

$$r'' = u(r, z),$$

where

$$u(r, z) = -\frac{er}{8mU} H^2(z).$$

By using the formulae (4) and (5) with r in place of z and time derivatives replaced by differentiations with respect to z the equation may be integrated immediately. Table 5 shows the computations which give the path of a 500 ev.

Table 5. Computation of an electron path using the paraxial equation

z (cm.)	r''	r (cm.)	z (cm.)	r''	r (cm.)
−12	0·0000	1·1660	0	−0·0207	1·7268
−11	0·0000	1·2240	1	−0·0185	1·7102
−10	0·0000	1·2820	2	−0·0142	1·6751
−9	−0·0002	1·3400	3	−0·0096	1·6251
−8	−0·0004	1·3978	4	−0·0055	1·5666
−7	−0·0007	1·4552	5	−0·0029	1·5018
−6	−0·0015	1·5119	6	−0·0014	1·4342
−5	−0·0030	1·5669	7	−0·0007	1·3654
−4	−0·0057	1·6190	8	−0·0003	1·2948
−3	−0·0098	1·6651	9	−0·0002	1·2252
−2	−0·0144	1·7014	10	0·0000	1·1546
−1	−0·0187	1·7236	11	0·0000	1·0844

electron through the field of figure 2, with $H_0 = 16{\cdot}5$ oersteds, the initial slope being $\tan\theta = 0{\cdot}058$. It is evident that a minimum amount of computation is necessary, and the method may be profitably compared with that due to Siday.

§ 4. EXPERIMENTAL TRACING OF ELECTRON PENCILS WITH A FLUORESCENT TARGET

To provide a comparison with the computational method, the paths were also obtained by experiment, the method being an adaptation of that used by Klemperer and Wright (1939) to investigate electrostatic lenses. An electron beam projected from a cathode system or electron gun passes through a "pepperpot diaphragm" which produces a number of fine pencils. A transparent fluorescent target which is normal to the axis of the lens is moved step by step along the axis and the fluorescent spots produced by the electron pencils are observed from the back of the target by means of a measuring microscope. The electron gun with "pepperpot" and sliding target are enclosed in an evacuated glass tube the interior of which is shielded electrostatically by a wire gauze. The effect of extraneous magnetic fields is eliminated by Helmholtz coils. The lens coil surrounds the glass tube and is arranged to be strictly co-axial with the gun. The axis is marked by a central pencil which is adjusted to have least deflection when the current through the coil is reversed. In the tracing of a given pencil,

measurements are always taken with a direct and a reversed field, since it proves to be quite difficult to reach complete agreement in the two cases.

There are two important differences between the procedures to be adopted for electrostatic and for magnetic lenses. In the former case (restricting our attention to non-skew rays) the electron path is a plane curve, so that it is only necessary to measure two co-ordinates to trace the path. In the case of the magnetic lens, however, owing to the gyro-action of the field, the path is a twisted curve and it is necessary to measure two co-ordinates x and y in a plane normal to the axis as well as the distance z along the axis itself. For this purpose, a cross-scale providing a metric in the x-y plane is arranged in the eyepiece of the measuring microscope and the z co-ordinate is measured by a pointer attached to the moving system. The second difference arises from the fact that the path can only be measured outside the electrostatic lens, since a fluorescent target placed inside the lens would disturb the electric field distribution. In the case of the magnetic lens, however, there is no disturbance, and the target may be moved through the whole field of the lens, thus allowing the full path to be traced. This possibility has not been used in earlier investigations. Becker and Wallraff (1938-40) have obtained many interesting results by ray-tracing through magnetic lenses with a fluorescent target, but they confined their attention to the rectilinear path which results after the electron has traversed the lens.

§ 5. COMPARISON OF SOME CALCULATED AND EXPERIMENTAL RESULTS

The first example relates to four 500-ev. electrons which pass through the field of figure 2 with $H_0 = 16{\cdot}5$ oersteds. Initially and finally the trajectories

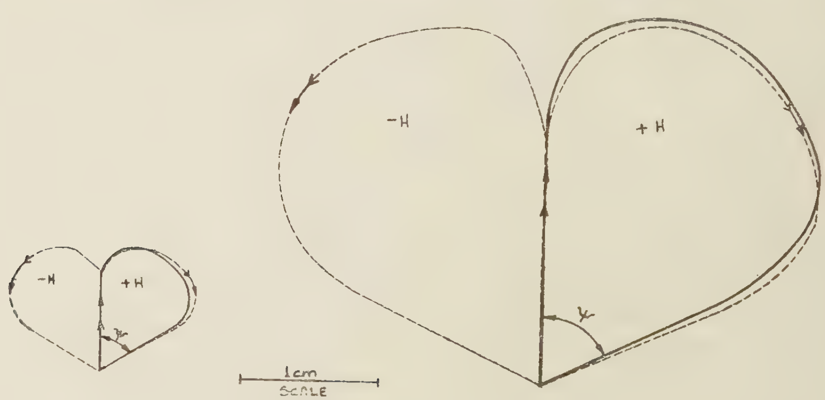

Figure 4. Tracing of initially homocentric bundle of 500-volt electrons through equatorial plane. $H_0 = 16{\cdot}5$ oersteds.
------- Experimental curves. ——— Computed curves.

are linear, the initial slopes being $\tan\theta$, and they first intersect the axis at a distance of 32 cm. from the centre of the lens.* Figure 3 shows the paths in the meridional plane, while in figure 4 the paths in the equatorial plane are drawn

* Table 4 relates to the pencil for which $\tan\theta = 0{\cdot}101$.

for the values 0·034 and 0·101 of $\tan\theta$, the notation $+H$ and $-H$ referring, in the experimental case, to the results obtained by reversing the current in the lens coils. It will be noticed that there is close agreement between the results obtained in the experimental and computational methods; in particular, between the total angle of rotation (ψ) of the meridional plane and the distance (z_f) between the lens centre and the point where a pencil intersects the axis after traversing the lens. This is shown in the following table:—

	ψ_0		z_f (cm.)		
$\tan\theta$	(i)	(ii)	(i)	(ii)	(iii)
0·034	59	60	27·0	23·8	26·8
0·058	66	61	23·2	22·2	26·4
0·101	67	63	18·0	18·4	26·5
0·125	67	65	13·3	15·3	26·4

(i) experimental values; (ii) computed values (general equations); (iii) computed values (paraxial equation).

As a further example, there is shown in figure 5 a tracing of pencils which are initially parallel to the axis and pass through the field of figure 2 with $H_0=20$ oersteds. The experimental mid-focal lengths are plotted against the semi-apertures, and in this way the spherical aberration curve, shown in figure 6, is obtained. The corresponding aberration curve obtained by the computational method is also shown, and it will be seen that the curves are in good agreement, although the latter is shifted towards the shorter focal lengths. The extrapolated intersections of the curves with the z-axis give the mid-focal lengths z_f, which, together with the focal lengths f, are compared in the following table:—

	Method			
	Experiment	Computation	Glaser	Busch
z_f (cm.)	10·4	10·1	9·6	8·9
f (cm.)	10·9	10·6	10·0	8·9

Here the values of z_f and f, according to Glaser (1941, p. 289), are given by

$$z_f = a\cot\frac{\pi}{\sqrt{1+\kappa^2}}, \qquad f = a\,\mathrm{cosec}\,\frac{\pi}{\sqrt{1+\kappa^2}},$$

where $\kappa^2 = \dfrac{eH_0{}^2a^2}{8mU}$, and the value of f, according to Busch, is given by the well-known formula

$$\frac{1}{f} = \frac{e}{8mU}\int_{-\infty}^{\infty} H^2(z)\,dz.$$

The mid-focal lengths and image distances obtained experimentally are quite generally found to be greater than those obtained by computation in the examples studied. This may be partly due to space-charge effects, but a more important factor is the effect of the finite size of the search coil leading to experimental values of $H_z(r,z)$, which are *too high*. This may be seen as follows. Suppose the inner and outer radii of the search coil are a and b, and that the coil centre is at the point (r_0, z_0). We consider first the case where $a<b\ll r_0$. If

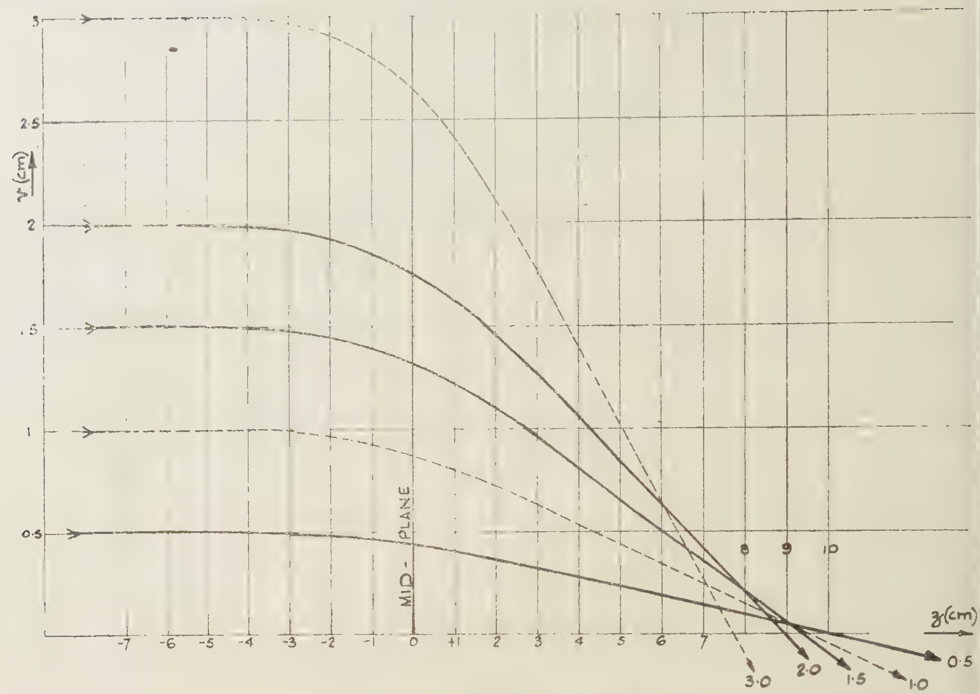

Figure 5. Tracing of 500-volt electron initially parallel to the axis. $H_0 = 20$ oersteds.

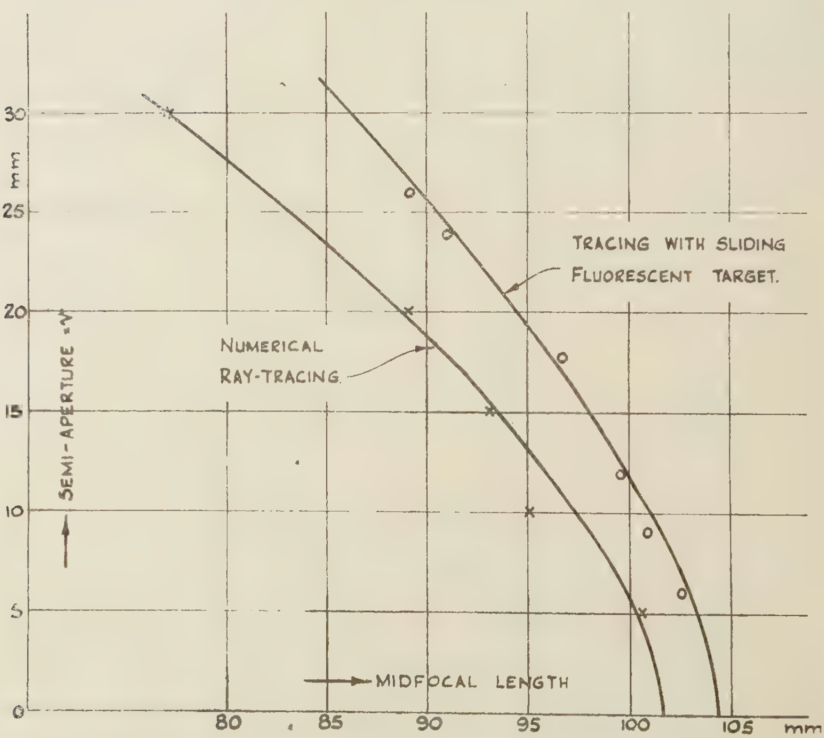

Figure 6. Spherical aberration curve for parallel electrons of 500 ev. $H_0 = 20$ oersteds.

$\bar{H}=\bar{H}_z(r_0, z_0)$ is the field strength measured as the mean flux threading the coil, the true value, as shown in the Appendix, is more closely given by

$$H=\bar{H}-\frac{3}{40}\left(\frac{b^5-a^5}{b^3-a^3}\right)\left\{\frac{1}{r_0}\frac{\partial \bar{H}}{\partial r_0}+\frac{\partial^2 \bar{H}}{\partial r_0{}^2}\right\}. \qquad \ldots\ldots(6)$$

Now an examination of figure 2 shows that $\partial\bar{H}/\partial r_0$ and $\partial^2\bar{H}/\partial r_0{}^2$ are positive in the stronger parts of the field (roughly for values of z_0 less than 2·5 cm.), so that from (6) it follows that $H<\bar{H}$.

The second case occurs when the search coil is used in the neighbourhood of the axis. In this case, if $\bar{H}=\bar{H}_z(0, z_0)$ is the measured axial field-strength, the value of $H_z(r_0, z_0)$ may be obtained from

$$H=\bar{H}+\left\{\frac{r_0{}^2}{4}+\frac{3}{40}\left(\frac{b^5-a^5}{b^3-a^3}\right)\right\}\frac{\partial^2 \bar{H}}{\partial z_0{}^2}. \qquad \ldots\ldots(7)$$

A study of figure 2 reveals that $\partial^2\bar{H}/\partial z_0{}^2$ is negative in the strong part of the field, for at the mid-plane we clearly have $\partial^2\bar{H}/\partial z_0{}^2<0$, since $H_z(r, z)$ is a maximum there and $\partial^2\bar{H}/\partial z_0{}^2=0$ at the inflexion, which occurs approximately when $z=4$ cm. In view of this it follows from (6) that we have $H<\bar{H}$.

Combining the two cases just examined, we see that the effect of a finite size of mesh coil is such that the field of figure 2 is stronger than the actual lens field. Consequently the values of the mid-focal length and the image distances derived from figure 2 (by the computational method) are smaller than those obtained experimentally. A higher accuracy would have been obtained by basing the computations upon the corrected field-distribution given by (6) and (7), instead of upon the uncorrected values of H given in figure 2. We have, however, refrained from doing so because, for the purpose of demonstrating the ray-tracing method, the accuracy of the field values of figure 2 is sufficient. On the other hand, the use of the corrected values may prove to be essential if small magnetic lenses have to be studied, the diameter of which may be only a few times larger than that of the smallest available search coil.

The accuracy of tracing with the fluorescent target is mainly limited by (1) spurious magnetic fields external to the system and departure of the system from strict axial symmetry; (2) the limiting finite diameter to which an electron pencil can be reduced. The use of Helmholtz coils and the reversing of current mentioned in §4 reduces the effects of (1) to a minimum. The accuracy of the reading of the x and y co-ordinates depends on (2), but with an adequate cathode system and 0·1-mm. apertures in the pepperpot diaphragm, the diameter of a spot may be made a fraction of a millimetre, even at the largest distances between pepperpot and target. Such a diameter is small compared with the semi-apertures of the rays shown in figures 4 to 6.

Regarding the computational method, errors due to approximation in the mathematical processes are negligible (as a study of table 4 will show) compared with the errors that arise in determining, by experimental field plotting, the field-distribution curves of figure 2, on which the computational method is based. The accuracy of the field plotting has a natural limit in the dimensions to which the search coil can be reduced without making the induced current too small for accurate measurement, although here a certain improvement may

be obtained by using the above formulae for correcting the measured values of the field. The ratio of the diameter of the search coil to that of the lens was less than 4 % in the example studied, so that the results obtained by the computational and experimental methods are probably of about equal accuracy.

APPENDIX

The field strength of H, as measured by the search coil, is strictly correct only when the field is homogeneous. For inhomogeneous fields, however, the correct values of H may be derived from the measured values by means of formulae which we now proceed to derive. Suppose the search coil is multi-layered, having t turns per layer, and that the inner and outer radii are a and b respectively. Let the centre of the coil be at the point (r_0, z_0), and let this point be taken as the origin of a local polar co-ordinate system (ρ, θ) in the plane of the cross-section of the coil. Suppose there are several layers between the outer and inner walls of the search coil, so that we can effectively replace a summation over the layers by an integration with respect to ϵ in equation (8). The value of H_z at the point (ρ, θ) is $H_z(\xi, z_0)$, where $\xi^2 = r_0^2 + \rho^2 + 2\rho r_0 \cos\theta$, so that the flux dN through an annulus of radius $d\rho$ is given by

$$dN = t\int_0^{2\pi} H_z(\xi, z_0)\rho\, d\theta\, d\rho.$$

It follows that the total flux through the coil is $N(r_0, z_0)$, where

$$N(r_0, z_0) = \frac{t}{b-a}\int_a^b\int_0^\epsilon\int_0^{2\pi} H_z(\xi, z_0)\rho\, d\theta\, d\rho\, d\epsilon. \qquad \ldots\ldots(8)$$

There are now two cases to consider. In the first case, suppose the coil is well removed from the axis so that $a < b \ll r_0$. Then, since $a < \epsilon < b$, it follows that $\rho r \ll r_0$, so that, neglecting ρ^3 and higher powers, we have

$$\xi = (r_0^2 + \rho^2 + 2\rho r_0 \cos\theta)^{1/2} \approx r_0 + \eta,$$

where $$\eta = \rho\cos\theta + \frac{\rho^2}{2r_0}\sin^2\theta.$$

Also $$H_z(\xi, z_0) = H_z(r_0 + \eta, z_0) \approx H + \eta\frac{\partial H}{\partial r_0} + \tfrac{1}{2}\eta^2\frac{\partial^2 H}{\partial r_0^2},$$

where $H = H_z(r_0, z_0)$. Hence the expression (8) for the flux becomes

$$N(r_0, z_0) = \frac{t}{b-a}\int_a^b\int_0^\epsilon\int_0^{2\pi}\left\{H + \eta\frac{\partial H}{\partial r_0} + \tfrac{1}{2}\eta^2\frac{\partial^2 H}{\partial r_0^2}\right\}\rho\, d\theta\, d\rho\, d\epsilon$$

$$= \frac{\pi t}{b-a}\left[\frac{H}{3}(b^3 - a^3) + \frac{(b^5 - a^5)}{40}\left\{\frac{1}{r_0}\frac{\partial H}{\partial r_0} + \frac{\partial^2 H}{\partial r_0^2}\right\}\right].$$

The measured value $\bar{H}$ of the field strength is obtained by taking only the first term of this formula, that is

$$N(r_0, z_0) = \frac{\pi t}{3}\left(\frac{b^3 - a^3}{b-a}\right)\bar{H}.$$

Hence the true value is given, approximately, by

$$H = \bar{H} - \frac{3}{40}\left(\frac{b^5 - a^5}{b^3 - a^3}\right)\left\{\frac{1}{r_0}\frac{\partial \bar{H}}{\partial r_0} + \frac{\partial^2 \bar{H}}{\partial r_0^2}\right\}.$$

The second case occurs when the search coil is used in the neighbourhood of the axis. Formula (8) still holds, but we use the expansion

$$H(\xi, z_0) \approx H - \frac{\xi^2}{4}\frac{\partial^2 H}{\partial z_0{}^2},$$

where H now stands for $H_z(0, z_0)$. We have

$$N(r_0, z_0) = \frac{t}{b-a}\int_a^b \int_0^\epsilon \int_0^{2\pi} \left\{ H - \tfrac{1}{4}(r_0{}^2 + \rho^2 + 2\rho r_0 \cos\theta)\frac{\partial^2 H}{\partial z_0{}^2} \right\} \rho\, d\theta\, d\rho\, d\epsilon$$

$$= \frac{\pi t}{b-a}\left[\frac{H}{3}(b^3 - a^3) - \tfrac{1}{4}\frac{\partial^2 H}{\partial z_0{}^2}\left\{\frac{b^5 - a^5}{10} + \frac{r_0{}^2(b^3 - a^3)}{3}\right\}\right].$$

As before, the measured value $\bar{H}$ is obtained by taking only the first term, so that the true value H is given approximately by

$$H = \bar{H} + \left\{\frac{r_0{}^2}{4} + \frac{3}{40}\left(\frac{b^5 - a^5}{b^3 - a^3}\right)\right\}\frac{\partial^2 \bar{H}}{\partial z_0{}^2}.$$

ACKNOWLEDGMENTS

The above research was carried out in the Laboratories of Electric and Musical Industries, Ltd., and acknowledgments are due to Mr. I. Shoenberg, Director of Research, and Mr. G. E. Condliffe. Thanks are also due to Dr. H. Miller for his co-operation in the early development of the experimental methods, and to Miss J. M. Charlton for carrying out the computations.

Note added in proof. Since the above results were submitted for publication, two papers dealing with related topics have appeared. In the first paper (K. Schlesinger, *Proc. Inst. Radio. Engrs.* **32** (1944), p. 483) a method has been given for obtaining optical constants of electron lenses. It is claimed that the trajectory problem is avoided, but this is not really the case since the method is essentially the same as an existing one based on the paraxial equation (see, for instance, L. M. Myers, *Electron Optics* (London, 1939), p. 176).

In the second paper (M. Deutsch, L. G. Elliott and R. D. Evans, *Rev. Sci. Instrum.* **15** (1944), p. 178), a method for calculating electron paths is given, starting with the equations of motion in the form given in our paper. The vector potential A is, however, calculated from the dimensions of the coil by, firstly, computing the field strength $H(0, z)$ on the axis, and then using the well known series expansion for A. This method is commendable, but it does not appear to be one of very general application. It is not, for example, applicable when the coil has iron pole pieces; moreover, the error introduced by the approximating processes is not so readily estimated as in our method. For the results obtained, the method also seems to involve more computation.

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DISCUSSION

on paper by A. J. Philpot, *Proc. Phys. Soc.* **56**, 268 (1944), entitled "Physics and the scientific instrument industry".

Mr. C. W. Hansel. Mr. Philpot makes a remark of great significance and well worthy of study when he says : " Today the schoolboy acquires a knowledge of physics almost equivalent to that required for a pass degree a generation ago ", and when he refers to the prevailing tendency for immature and inexperienced schoolboys to grapple with such problems as those of atomic and nuclear physics rather than with the " more prosaic physics of yesterday ".

I do not think that I am alone in my view that the excusable mistake of a generation ago of covering too much of the factual content of theoretical physics and a course of cast-iron routine in practical physics has been intensified today by the introduction of general science, for which there is no excuse. Has not the teaching of science deteriorated when a subject such as general science finds favour with the Science Masters' Association and the Board of Education ? General science sacrifices individual experimental work to mass demonstration and a piling up in the mind of the factual content of science. It isolates physics from mathematics and subordinates—indeed almost eliminates—the technique of science in order to cover an encyclopaedic range of fact. Quoting T. H. Huxley : " If scientific education is to be dealt with as mere bookwork, it will be better not to attempt it, but to stick to the Latin Grammar, which makes no pretence to be anything but bookwork ". Charles Kingsley has truly said : " . . . . after all, an experiment is worth very little to you, unless you perform it yourself, ask questions about it, or vary it a little to solve difficulties which arise in your mind." Also John Ruskin : " There used to be, thirty years ago, a little rivulet of the Wandel, about an inch deep, which ran over the carriage-road and under a foot-bridge just under the last chalk hill near Croydon. Alas ! men came and went ; and it—did *not* go on for ever. It has long since been bricked over by the parish authorities ; but there was more education in that stream with its minnows than you could get out of a hundred pounds spent yearly in the parish schools, even though you were to spend every farthing of it in teaching the nature of oxygen and hydrogen, and the names, and rate per minute, of all the rivers in Asia and America."

OBITUARY NOTICES

SIR CHARLES BOYS, F.R.S.

Charles Vernon Boys was born at the Rectory of Wing, Uppingham, in Rutland, on 15 March 1855 ; he was the eighth child in a family of nine—five sons and four daughters—six of whom married, and one, a boy, born in 1854, died at birth.

" C.V.B."—so to call him—came of a good stock. His father, the Rev. Charles Boys, Rector of Wing, was a man of scientific tastes, a skilled carpenter and ship model-maker, and occasional compounder of home-made fireworks. His mother, *née* Caroline Goodrich Dobbie (b. 1816), was the fifth child of Captain William Hugh Dobbie, R.N. Caroline's elder brother, W. H. Dobbie, C.I.E., was the father of Sir William G. S. Dobbie, now celebrated as the heroic Governor of Malta during its long siege by German aircraft. C.V.B.'s paternal grandfather was Captain C. W. Boys, R.N.—another heroic figure, who lost a leg, as midshipman in the ' Royal George ', during Lord Howe's victory off Cape Finisterre on the " Glorious First of June " 1794. This hardy officer continued to serve in the Navy, with one leg, for 14 years, was promoted captain, and died at sea in 1808. C.V.B.'s paternal grandmother—the captain's wife—was Mary Vulliamy, daughter of the famous clockmaker (b. 1780, d. 1854), who was also noted as a very careful experimenter. Mary Vulliamy's brother was Lewis Vulliamy (b. 1791, d. 1871), the well-known architect, who designed the façade of the Royal Institution in Albemarle Street, Dorchester House, the Law Institute, etc., and was Royal Academy Gold Medallist in 1813.

Even before he was five years old, C.V.B. gave evidence of exceptional mental alertness and clarity. His sister, Caroline Egerton, relates that while C.V.B. was " still in petticoats ", he was present when someone set the puzzle : " If a salmon weighs 7 lbs. and half its own weight, how much does it weigh ? " While there was much arguing as to the answer, C.V.B. was observed sitting apart and muttering to himself : " How stupid they all are ; what can you add to a half but a half ? 'Course it's fourteen. How stupid they are ".

His boyish interests were specialized : he did not care for books, or games, or hobbies; he did not sing—as the rest of his family did—but he had a good ear for music.

He quickly achieved skill in handicraft, and enjoyed making elaborate fireworks. Miss Alice Bell, one of C.V.B.'s earliest friends, recalls that one day when she and her sister were winding silkworm cocoons, C.V.B. came in and watched the operation. A few days later he came again, bringing " a wooden frame which held a wheel, absolutely the right thing for winding cocoons ". Years afterwards she showed C.V.B. this apparatus, and he pointed out that it was made of five different kinds of wood. It is clear that, even in his boyhood, he had the instinct for thinking functionally, and for providing for the physical and mechanical welfare of every element of the apparatus he constructed.

C.V.B. had a great affection for his father, who encouraged his son's scientific tastes and worked with him when he came home to the Rectory.

In 1869 C.V.B., then 14 years old, went to Marlborough College (C.I. House), and, in his second year, came under the influence of G. F. Rodwell, F.R.S., F.C.S., F.R.A.S., who joined the staff as a science master—having previously taught at Clifton, where the present Sir James Swinburne, F.R.S., was one of his pupils. Rodwell's coming determined C.V.B.'s course in life. Nineteen years later, when he wrote his book on Soap Bubbles, he dedicated it to Rodwell in these words : " To G. F. Rodwell, the first Science Master appointed at Marlborough College, this book is dedicated by the author as a token of esteem and gratitude, and in the hope that it may excite in a few young people some small fraction of the interest and enthusiasm which his advent and his lectures awakened in the author, upon whom the light of Science then shone for the first time ".

C.V.B.'s only son—Geoffrey Vernon Boys, M.I.E.E., M.I.Mech.E., Secretary to the Institution of Naval Architects—records that after his father had received the Royal and Rumford Medals, " great masses of gold " (they must have weighed together over 2 lbs.

avoirdupois), he had them melted down and sold, and gave the proceeds to Marlborough College for the institution of an annual prize for science at the school. The present Headmaster of Marlborough, Mr. F. M. Heywood, reports that there are now two Boys' prizes offered annually for good work in Science, as the result of this donation—a Senior prize, which is open, and a Junior prize, limited to boys passing direct from the Middle School into the Science Fifth. We must add gratitude to the tale of C.V.B.'s virtues.

In December 1872, at the age of 17, he left Marlborough and entered the Royal School of Mines—then lately transferred to South Kensington from its original home, the " Government School of Mines and of Sciences applied to the Arts " at the Museum of Practical Geology, Jermyn Street, founded in 1851. As to this period, Dr. H. J. T. Ellingham has supplied the following facts : In 1876, C.V.B. was awarded the diploma of Associateship of the Royal School of Mines, of which the Professor of Physics was Frederick Guthrie and of Applied Mechanics Thomas Goodeve.

During the period 1876–81, C.V.B. was sometime private assistant to Dr. Percy (then working at Jermyn Street) and afterwards to Dr. Guthrie at South Kensington.

In 1880, Guthrie—the Founder of the Physical Society—granted him a Life Membership of the Society, of which he was Demonstrator for about ten years.

In 1881, C.V.B. was Demonstrator in Physics (under Guthrie) in the Normal School of Science (renamed The Royal College in 1890), where he remained until 1897. During this time, C.V.B. worked in the Huxley Buildings, or in the temporary laboratories in the Exhibition Buildings—on the opposite side of Exhibition Road—which were secured in 1886. In 1883–84, all the researches on a magneto-electric phenomenon " were communicated to the Physical Society, which is privileged by the grace of the Lords of the Committee (on Education) to hold its meetings in the Physical Lecture Room on certain Saturday afternoons "—a great privilege in those unscientific days. In 1885–86, research " could only be carried on with difficulty ", owing to the increased number of students (42 regular and 13 occasional), but C.V.B. described, before the Physical Society, his machine for the solution of equations of any degree, and made experiments on simple methods of determining wave-lengths of light. In 1886, Guthrie died, and C.V.B. took charge of the department until the appointment of Rücker (afterwards Sir Arthur Rücker) as professor in 1887. In this year, C.V.B. published his work " On the Radio-micrometer, a new instrument for measuring the most feeble radiation ", " On the production and properties and some suggested uses of the finest threads " and " On an addition to Bunsen's Ice Calorimeter " ; he also presented a Note, on the tenacity of spun glass, by his pupils E. Gibson and R. A. Gregory (now Sir Richard Gregory, F.R.S.).

In 1882, when C.V.B. was demonstrator in Physics, he had bought his " Otto " di-cycle. This machine—invented by E. C. F. Otto—brother of the inventor of the Otto gas engine—consisted of two 56-in. cycle wheels mounted on a horizontal axis which carried the saddle ; a double-cranked shaft, with toe-capped pedals, was carried by telescopic arms at either end of the horizontal axle. Transmission from the cranked shaft to the cycle wheels was made by a pair of flexible metal belts passing over flanged, rubber-lined, pulley wheels at either end of the crank-shaft and on the inner sides of the cycle wheels. Either belt could be caused to slip—more or less—by varying the length of one or other of the telescopic arms—viz., by twisting either of two spade handles carried on the horizontal axle. The machine then turned in the direction of the slipping transmission. Somersaulting backwards was prevented by a rigid, backward and downward curved arm, at the centre of the horizontal axle, which carried a small (emergency) rubber-covered wheel ; somersaulting forward was prevented by the rider's instinct of self-preservation—he pushed himself back by pushing the pedals forward !

C.V.B. completely mastered this precarious device, and at a Royal Institution Lecture in 1884 demonstrated the feat of spinning the Otto " on its own ground ". This was done by connecting the two wheels alternately to the cranked shaft, and moving the pedals only a very little each way ; later on, the Otto was replaced by a motor bicycle.

In 1887–88, C.V.B. published papers " On the Optical Demonstration of Electric Stress " (with Rücker) and " Experiments with Soap Bubbles ". In 1888–89, C.V.B.—now Assistant Professor—published " The Radio Micrometer ", " On the Cavendish Experiment ", and " Quartz as an Insulator ". He was elected F.R.S. in 1888. The records of the R.C.S. note that C.V.B. " served on Juries at the Inventions Exhibition in London

and at the last Paris Exhibition, when he was made Officier de l'Instruction Publique. The International Inventions Exhibition was held in 1885. The "last Paris Exhibition" was presumably the Exposition Universelle of 1889. There was also an Exposition Universelle Internationale in Paris in 1900, at which C.V.B. acted as foreign Juror in "Groupe III. Classe 15. Instruments de précision". At one or other of these, C.V.B. exhibited his constant-of-gravitation apparatus, his radiomicrometer, and his production of quartz fibres.

From about the year 1893 onwards, C.V.B. developed a lucrative practice as expert witness in patent cases. He was not much interested in patent law, nor in the conduct of the case, but—as Sir James Swinburne describes it—"what absorbed his attention was methods of explaining mechanical points clearly to the Court. . . . . He acted once as an Assistant on the Bench to Ridley, in a case about a weaving machine (improvements in Dobby looms), and put the matter so clearly that he practically settled the case".

Fom 1895 onwards, for 20 years or more, C.V.B. continued to develop his practice as expert witness. Particulars of this work have been supplied by his friend, Mr. Horatio Ballantyne, and Sir James Swinburne, both very successful experts in the same line—also by Messrs. J. B. Purchase (now J. B. Purchase and Clark), the solicitors to the Dunlop Company, Messrs. Bristows, Cooke and Carpmael, and others.

Between 1895 and 1900, C.V.B. was one of the team of witnesses retained by the Dunlop Company, under the leadership of Sir John Fletcher Moulton, K.C., F.R.S. (afterwards Lord Moulton), in the series of hard-fought actions which established their hold of the cycle- and motor-tyre markets. Mr. Ballantyne's list covers the following inventions and processes: Smokeless powder, the electrolytic production of caustic soda, gramophone horns, the alumino-thermal process of welding and joining rails, etc., etching metal plates by acid spraying, grinding balls for ball-bearings, squirted and fine-drawn tungsten filaments for electric lamps, milk-drying by means of steam-heated rollers, compressed-air-driven rotary engines for coal-cutting, sealing tungsten leading-in wires for electric lamps, making and stiffening the fore-parts of boot and shoe "uppers", and the continuous production of cast strips for printers' "leads" and "rules". To these may be added several other boot- and shoe-making machinery patents—the petition for prolongation of the patent for the Fleming valve, the Haskell golf-ball, gyroscopic compass, and wireless telegraphy (Marconi) patents. It is evident that his work as expert witness covered a very wide range of highly technical subjects, all of which he clarified.

On resigning his Assistant Professorship in 1897, C.V.B. was appointed a Metropolitan Gas Referee, with offices at 66 Victoria Street, where he eventually secured additional rooms for his private use as workshop and laboratory. This post he held till 1920, when he became one of the three Gas Referees provided for the whole country by the Gas Act of that year. Major Claud G. Hyde, F.R.I.C., who was for some time C.V.B.'s personal assistant, says: "It was his practice to do his thinking and draughting at his office in Victoria Street during the week, and to carry out his constructional work at the week-ends at his home at St. Mary Bourne, where he had turned his drawing-room into a workshop. He would return to London on Monday or Tuesday morning, carrying the results of his week-end labours in a fish-basket, and he would show them to the first person he met in the building, often the postman or a clerk. . . . . He had a grand sense of humour and a charm of manner that endeared him to the whole of his staff, the most humble members of which were at some time or other called in to share his jokes". He continued to act as Gas Referee till 1939, when he was 83.

Boys, according to Dr. J. S. G. Thomas—who was associated with him in gas calorimetry from about 1913—revolutionized calorimetric design in all its aspects, and did much to place the supply of gas to the public on a strictly scientific basis.

Before his appointment as Gas Referee, when his stipend as Assistant Professor was far from adequate to his needs, C.V.B. found time to lecture "all over the country" at fees of about £10 10*s*. 0*d*. per lecture: these lectures were very popular.

In 1898 (the year after his appointment as Gas Referee) C.V.B. became one of the Founder Members of the Automobile Club (afterwards the Royal Automobile Club) and was associated with the encouragement of the motor industry from its earliest years. He was a member of the Club Committee, also of the Technical and Competitions Committees, and freely gave his scientific and mechanical advice, especially in connection

with the various trials and competitions held by the club. He was Vice-Chairman 1907–8 and elected Vice-President in May 1931.

About the year 1900, C.V.B.—now aged 45—devised his revolving lens camera for photographing lightning flashes—to which he gives pride of place in his biographic entries in *Who's Who*. In 1903 he was President of Section A of the British Association at their meeting at Southport. In 1918 C.V.B. delivered his Presidential Address to the Physical Society, on the experimental art, the use of tools and materials, and the development of manual skill.

Between 1924 and 1934 his time was mainly employed in experiments on calorimeters. In 1928 he was the guest of Mr. A. L. Loomis at his country house and laboratory at Tuxedo Park, U.S.A., and while there he made his first successful experiments in photographing a lightning flash with his revolving lens camera (which he had planned in 1900). The Loomis Laboratory was probably unique in scientific history. It consisted of a large country house in delightful surroundings, owned by Mr. A. L. Loomis, a wealthy banker of scientific tastes and a passion for the advancement of science. The house itself, which the present writer was privileged to visit, was divided into a series of apartments where visiting scientists, with their wives and families, were entertained as Mr. Loomis's guests during vacation or other free time. The massively built basements had been converted into laboratories, supplied with the most modern apparatus, where the visiting scientists could carry out their chosen researches, untrammelled by economic limitations. Their wives and families were provided with tennis courts and other games, and ample playing grounds for their children ; an English officer and his wife acted as resident host and hostess. There must be many English country houses which could be adapted for similar purposes if the necessary vision and capital were forthcoming !

About 1933, C.V.B. was operated on for cataract, and eventually lost the sight of one eye completely.

In 1935, in his 80th year, he received the well-deserved honour of Knighthood.

His method of constructing graphical representation of solid objects, was published in *J. Sci. Instrum.* May 1942. His latest work, on the design and construction of an elliptograph, appeared in these *Proceedings* (**55**, p. 411).

As to C.V.B.'s work in pure physics, no attempt will be made here to describe the many subjects he investigated, or the results that he achieved. A Notice of C.V.B. for the Royal Society has been written by Lord Rayleigh, in which this aspect of his work is fully dealt with. The present writer has been privileged to see this notice, and has arranged to deal more particularly with C.V.B.'s other interests, so that the two notices read together and may present as little duplication as possible.

C.V.B. was pre-eminently practical in his outlook—his work as expert witness brought him in contact with many industries—and it is, therefore, not surprising that he was a fertile inventor. Search at the Patent Office discloses that during 58 years—1881 to 1939—C.V.B. filed 87 applications for letters patent, of which 53 were completed—the remainder having either been abandoned before completion or found to have been anticipated.

The completed patents cover a wide range—they may be roughly classed as follows :—

Measuring devices : Measuring mechanical power (1881). Electrical meters (1881, 1882, 1883). Engine power meter (1882). Measuring and registering transmission by belts (1884). Electrometers (1891). Weighing apparatus (1927). Thermometers (1927). Altitude and azimuth for balloons (1929).

Mechanical devices : Equalizing drive in clocks (1929). Differential driving gear for velocipedes (1883). Cyclometers (1884, 1886). Driving and steering velocipedes (1886). Speed gears for motor vehicles (1904). Motor vehicle wheels (1916). Converting rotary into reciprocating motion (1907). Tool-grinding (1900). Pneumatic hammers (1907).

Gas and air and steam : Dust separation from air (1907). Air pumps (1907). Cleansing steam (1885). Fluid operation pistons (1907). Elastic fluid engine (1908). Reducing and preventing noise of escaping gas (1886). Gas calorimeters (1921, 1923). Measuring and delivering gas (1923). Sampling and delivering gas (1925, 1936). Delivering water, etc., in measured quantities (1936). Liquid flow gas calorimeter (1939)—C.V.B.'s last patent to be completed.

Electricity : Secondary batteries (1882). Incandescent lamps (1882). Insulators (1889). Electric safety plugs or fuses (1883). Wireless telegraphy receiving instrument (1905).

Liquid films, etc. : Liquid films (1911). Production of bubbles (1912). Apparatus for treating liquid films (1912).

Miscellaneous : Movable pictures or diagrams for advertising (1892). Welding rails (1902). Diabolo (1907, two patents). Wool-combing machine (1908, two patents). Retorts for heat treatment of fluids (1913).

C.V.B.'s character never apparently varied throughout his long life. His son thus describes him : " My father had no hobbies, seldom read books, novels or more serious works, but used to absorb himself in geometrical problems. He was very fond of cross-word puzzles ". Physically he was of light, athletic build, about 5 ft. 8 in. in height, and of Nordic type, with blonde hair which whitened in after-life—a round head, high forehead, and a very lively and normally happy expression. His movements were swift and accurate.

He was always interested in practical gardening, and his little book, *Weeds, Weeds, Weeds*, published in 1937, when he was 82, is typical of his methods.

He was precise and concise in everything he did—in his way of speaking, his manner of thinking and in his methods of planning his work and constructing his apparatus. After his death, his business papers were found to be in " apple-pie order ".

C.V.B. was by nature a solitary worker—his field of action was his laboratory at 66 Victoria Street. He was not a member of the Institute of Civil, or Mechanical, or Electrical Engineers, and never went near these places. He did little, if any, consulting engineering work. But he enjoyed the meetings of the Royal Society Club and the Physical Society Club—of which he was one of the earliest members—and the company of his scientific friends at the Savile Club and the Athenaeum. But though so compactly self-contained, he was outstandingly kind and generous to all who appeared to him to be in need of his help.

Sir Richard Gregory—whose work with E. Gibson (1886–87) has already been mentioned—says : " It was characteristic of Boys to do everything himself, and to make efficient use of the simplest instrumental equipment. It was equally characteristic of him to communicate to the Physical Society some observations made by two young students in his laboratory. Boys' influence upon all his students was by example rather than precept ".

There is, as usual, another side to this picture—namely, that drawn by H. G. Wells, in his *Experiment in Autobiography* (vol. i, pp. 206–18), to which some reference should be made.

Wells, after an inspiring year under Huxley at the Normal School of Science (afterwards renamed the Royal School of Science) found Guthrie " too slow ". As Wells put it : " He swept aside the idea that Physics is an experimental Science, and substituted a confused workshop training ". Wells was not interested, and rebelled vigorously. Boys, on the other hand, he found " too fast ". As he puts it : " Boys shot across my mind and vanished from my ken with a disconcerting suggestion that there was a whole dazzling universe of ideas, for which I did not possess the key ". Wells thought C.V.B. " one of the worst teachers who had ever turned his back upon a restive audience, messed about with the blackboard, galloped through an hour of talk, and bolted back to the apparatus in his private room ". C.V.B. became, as we know, a very successful and popular lecturer, but he was certainly not a born teacher, and it is likely enough that he also did not enjoy the lecture he was obliged to give on thermo-dynamics—to which Wells's criticisms refer.

Dr. Thomas (already referred to) was one of the first visitors invited to see C.V.B.'s completed recording gas calorimeter, prior to its publication. On this occasion, C.V.B. related that when he had just completed his first recording instrument—the gas meter of which was made of celluloid—a visitor pointed to it with his lighted cigar. The meter took fire, and " two years' work went up in flames ". The visitor enquired why the meter had been made of such a dangerous material ; to this C.V.B. replied that it was the visitor and his cigar, not the celluloid, that constituted the danger. C.V.B. then started again, and completed his second instrument.

Major C. E. S. Phillips records the following : " As a young man, I wrote to Boys

(not knowing him personally) on a surface tension problem, and remember that, much to my surprise and delight, he sent me a very long letter in reply, making everything clear ".

C.V.B.'s nephew, Hubert Egerton, similarly tells how he used to consult his uncle about patent matters relating to his own business, and always got ungrudging help—for which his uncle would never accept any form of monetary return. C.V.B.'s son also describes how, when his father's income as Assistant Professor, and later as Gas Referee, was still " a mere pittance ", C.V.B. contributed generously towards the stipend of a Curate at Wing to assist his then very aged father. There can be no doubt of his generosity and loyalty.

On the other hand, as Sir Richard Gregory records : he " did not care a rap about conventions, whether social or scientific ". His scientific unconventionality was probably all to the good ; but his social unconventionality had, as we shall see, a sad aftermath.

In 1892, when he was 37, C.V.B. married Marion Amelia Pollock, daughter of Henry Pollock ; they had two children—a son, Geoffrey Vernon, born 1893, and a daughter, Margaret Angela (afterwards Mrs. Malcolm Carruthers), born in 1896. G. V. Boys married, in 1933, Helen Florence Mary, daughter of Frank Gossling, and has a son, John Vernon, who, at the age of 10, shows promise of high intelligence.

The present writer had the privilege of meeting C.V.B. and his wife at their home, 27 The Grove, Boltons, about 1895–96, when they had been married three or four years. Even then the contrast of their characters was evident. C.V.B. was just himself. His wife was also just herself—attractive, bright, intelligent, eminently social—and an excellent hostess, but little equipped by nature and disposition to understand or sympathize with her husband's puck-like unconventionalities. Of these there are many instances on record. Lt.-Col. Mervyn O'Gorman writes : " He came to my office once to show me how he managed to appear all right (in a law case) when he had no time to ' tidy up '. His shirt was dirty. He cut a stud-hole in a piece of foolscap and slipped it under his tie and waistcoat : ' You see ? A perfectly clean shirt ! ' ".

C.V.B.'s natural instinct was to do things in his own way—to drink his tea out of a saucer if it wanted cooling, or to come to dinner in his working clothes if, after his day's work, he felt too tired to change them. These were obviously small things in themselves, but they were cumulative in their effects. Neither party could really understand the other's point of view, and in the end they parted—and C.V.B. obtained a divorce in 1910, eighteen years after his marriage. Mrs. Boys afterwards married A. R. Forsyth, F.R.S., and died in 1920. The truth is that C.V.B. was a constitutional bachelor, and remained so all his life.

Dr. W. Eccles, who knew him well, thus describes him : " He had one of the quickest minds known to me. He understood *anything* with speed and accuracy, even when the description given him was a poor one. He was remarkably business-like also—a rare thing in a devotee of Pure Science. Perhaps he was more objective than abstract—this would account for his difference from the typical Don ".

Sir James Swinburne (already referred to), who knew him for fifty years, says : " Everybody liked him, with his whimsical humour and sharp brain. No one could dislike him, and I never heard of anyone who did ".

Something must be said about C.V.B.'s delight in practical jokes. These were invariably ingenious, unexpected and harmless—unfortunately not many appear to have been recorded. Here are a few of them. About 1903, when C.V.B. was 50 and his son a schoolboy, C.V.B. is reported as filling india-rubber balloons with water and dropping them from a high window so as to fall just behind a walking victim. No harm was done, but the noise of the burst was " very peculiar ". His schoolboy son was not allowed to know of these paternal escapades, for fear of corrupting him ! Horatio Ballantyne tells how, being in Paris with C.V.B. in 1913 (conducting experiments in connection with electric lamps), they were dining quietly together after a hard day's work. C.V.B. suddenly burst into uncontrolled laughter—he had just remembered a practical joke which he and his colleague (both Assistant Professors at the R.C.S.) had played on Norman Lockyer some 23 years previously. Lockyer had set up a large battery of Grove cells which he used for his lectures. C.V.B. and his colleague took the platinum foil out of one of the cells and substituted a piece of tinfoil. The battery failed to operate, and when the

defective cell was spotted, it was thought (by Lockyer at least) that a new modification of platinum, soluble in nitric acid, had been discovered. Ballantyne also tells how Boys successfully protected his garden from cats by a fence of " parallel sloping and sagging wires, one above the other ", so that the cat, trying to climb over it, slithered down gently sideways on the sloping wires. Cats, he had observed, hated to be affronted or put in a ridiculous position. C.V.B. delighted in describing their discomfiture.

On 15 March 1935, C.V.B.'s 80th birthday was celebrated at the Royal Society Club—of which Lord Rayleigh has given a description in his notice for the Royal Society. A complimentary dinner was also given by the Physical Society Club, at which a poem, " Boys will be Boys "—composed by the present writer, and now reprinted (by request)—was sung to a tune composed by the author, who, with Dr. J. H. Brinkworth, each sang one line alternately, in order to save their breath and maintain the rhythm. C.V.B. was visibly pleased with this tribute, and had the poem inscribed on parchment by the well-known illuminator, Grayley Hewett, as a family heirloom.

C.V.B.'s mental powers and courage never failed, but his eyesight deteriorated seriously. In describing his elliptograph—in 1943, when he was 88—of which both the apparatus and the drawings were made by himself, he explained that the scheme of binocular vision had been a great handicap. The fact was that he used his delicate touch as much as his other senses. He retired to his home at St. Mary Bourne, Andover, where he died in 1944, fifteen days after his 89th birthday. He was without doubt one of the really great experimentalists ; he devoted his whole life to Science, and his latter days were made very happy by the realization that his work had been fully appreciated, and that he was honoured and beloved. The subjects to which he was devoted are recorded in the following verses:

To Sir Charles Vernon Boys on his Eightieth Birthday

Why does Sir C. V. Boys elect
To do the things we least expect,
And always choose a task that seems
More suited to the land of dreams :
A problem other men would shirk,
Yet solve the task and make it work
By means that *no one* else employs ?
The answer is : Boys will be Boys !

Why did his bold, untrammelled thoughts
Conceive the scheme of fusing quartz,
Using an arrow, as it fled,
To draw a microscopic thread,
And from the fusion to " unreel "
A gossamer more true than steel,
Which every Physicist enjoys ?
The fact is this : Boys will be Boys !

What made him buy an Otto bike—
Two wheels abreast—a thing to strike
Terror in any rider's soul,
Yet somehow manage to control
Its action and avoid a spill,
And—using his uncanny skill—
Delight in a precarious poise ?
Again we say : Boys will be Boys !

What made our friend so seeming rash
As to pursue a lightning flash
By lenses rapidly revolved,
And even get the problem solved—
Both of its speed and structure—by
A photograph " which cannot lie " ?
That gave a thrill that never cloys,
And showed us still, Boys will be Boys !

Why snatch a bullet in its flight,
Lit by a single spark so bright
That on a photographic plate
The fleeting shadow seemed to wait—
With wake and bow-wave primly set—
All posing for their silhouette—
And leave a picture of the noise ?
Because, of course, Boys will be Boys !

To weigh the earth—to check the Therm—
Explain the logarithmic term—
To build with bubbles, and maintain
The opal colours in their train !
These are his pleasures, these his ploys
(Where skill with mind and Truth alloys)
For which, in Science, as in Toys,
We thank our stars, Boys *will* be Boys !

R. A. S. PAGET.

EVAN JENKIN EVANS

I FIRST met E. J. Evans in October 1908, when he took up duty as a Demonstrator and Assistant Lecturer in Physics at the Victoria University of Manchester. I was one of a dozen new entrants into the Honours School of Physics, and by the end of the first week or two of term every one of that dozen had begun to realize that Evans was an exceptionally fine teacher and one of the most likable of men. At any time of stress with a difficult experiment or a recalcitrant piece of apparatus, the sight of his chubby face, with its friendly and slightly puzzled expression, was as welcome to the struggling student as that of an approaching sail to the ship-wrecked mariner ; there were very few students in the department who did not profit by, or sometimes trade upon, his boundless good nature. As time went on, and especially after I joined him on the teaching staff, I came to know him very well, and every year augmented my respect and liking for him as a teacher and friend.

He had a clear and accurate mind, trained in one of the best and most rigorous all-round courses in physics then available, that leading to the Associateship of the Royal College of Science, followed by a period of research under the distinguished spectroscopist Professor Alfred Fowler. He had an excellent mastery of classical physics, and was quick to assimilate modern developments. He was a gifted and conscientious—perhaps even over-conscientious—teacher, and it was obvious that he put an immense amount of nervous energy into his lecturing.

In the laboratory his manner with students was admirable, and he quickly gained their confidence. He never used sarcasm, and his attitude to students' enquiries was that of a fellow-seeker after truth, not that of pompous authority speaking down to ignorance. He was modest almost to a fault, and inclined to be too diffident even in expressing opinions on matters of which he had sound specialized knowledge. Manchester students were, however, generally shrewd enough to realize that Evans's habitually mild exterior covered a great deal of hidden power ; they soon learned to respect his authority, and even the most turbulent undergraduates hesitated to play any of the usual tricks upon him or in his lectures. Any who did transgress found out quickly enough that he was not quite so guileless as they had assumed him to be.

He devoted much time and energy to spectroscopic research of high quality, working long after normal laboratory hours. It was perhaps unfortunate for him at first that the interests and resources of the laboratory were almost exclusively concentrated upon problems in radioactivity, but he had his reward when spectroscopy became linked with nuclear physics, and he was able to make his well-known contribution to the problem of the origin of the Fowler and Pickering series of spectral lines.

Evans led the simplest and quietest of lives, and was obviously uninterested in matters of dress or personal adornment. He found for himself simple but comfortable lodgings within a hundred yards of the University buildings, and he seldom strayed, save for the purchase of books or cigarettes, far from the direct route between his lodgings and the laboratory. I probably owe a good deal of my knowledge of him from the circumstance that I happened to share two of his few interests outside his work. We were both devotees of the now obsolescent music-hall, and we had a common interest in ships and seafaring.

Almost every week we found time for one evening at the second house of a music-hall in an outlying district, where there was generally an excellent entertainment and where, if memory serves me, the cost of the best seats in stalls and dress circle was ninepence, and where an additional twopence or threepence (which we, in fact, never expended) procured a seat in a stage box. We occasionally visited the larger and more modern halls in the city's theatrical centre, but I think that on the whole we found them too glittering and sophisticated for our simple tastes. Evans hugely enjoyed these expeditions, which for weeks at a time were almost his sole relaxations from continuous overwork.

His passion for ships was less easily gratified in Manchester. It found some outlet in occasional visits to the Liverpool docks, which in those days were freely open to public inspection. Evans knew the docks from end to end, and was familiar with the funnel markings and house-flags of all the principal and many of the smaller lines. He would spend a thoroughly happy day along the waterfront, examining the liners in the various docks, and getting tremendously excited when he saw some vessel of an obscure line which only rarely used the port—very much in the spirit of a boy adding a new stamp

to his collection or completing a set of cigarette cards. Although it is over thirty years old, I still treasure the memory of a day we spent together in an exceptionally fruitful tour of the docks, followed by a return to Manchester, just in time for a particularly hilarious music-hall show, starring George Graves at the height of his form.

These, I know, are trivial recollections, but I think they are worth recording as little-known aspects of a man who, in spite of his modest and retiring disposition, played for many years a part which is probably greater than is generally realized in the training and building of Rutherford's school in Manchester. H. R. ROBINSON.

Professor Robinson has given above a pen picture of E. J. Evans that will be endorsed by all who knew him at Swansea. It only remains to give a more formal account of his career.

Evans, a native of Llanelly, graduated at the University College of Aberystwyth in 1902. He then proceeded to the Royal College of Science, graduating there in 1906, and remaining in South Kensington, in turn as demonstrator in astrophysics and in physics, until in 1908 he was appointed assistant lecturer in physics at Manchester. He did spectroscopic work in the late Prof. A. Fowler's laboratory and retained his interest in this branch of physics until his death.

At Manchester he worked on the absorption spectra of iodine and other substances at high temperatures. Then followed his most important work, that on the ionized helium series, for which he obtained his D.Sc. These series, of which some lines (e.g. the $\lambda 4686$ line and the Pickering ζ Puppis lines) were already known in celestial sources, had been produced in a laboratory source for the first time by Fowler, who used He tubes known to contain traces of H_2, and who attributed the series to H on the basis of purely empirical numerical relations. Bohr, who was then at Manchester, predicted that all the lines, which Fowler had arranged into four H series (including the Balmer) with the usual Rydberg constant $\boldsymbol{R}$, should really form two He^+ series with $4\boldsymbol{R}$ as the constant. At Bohr's suggestion Evans set about producing the series in tubes showing no trace of H, and he also improved the λ measurements so that a slight difference between the wave-lengths of the Balmer H lines and the alternate He^+ lines was put beyond doubt. Fowler, though loth to abandon the assignment to H, admitted that the weight of theoretical and experimental evidence was against him. He felt, I think, that if his original view was wrong, there was at least some satisfaction in reflecting that it was his former pupil Evans, backed by no less a genius than Bohr, who had proved it wrong.

In 1915 Evans became senior lecturer and in 1919 assistant director of the Manchester laboratories. He was also in charge of the department during Rutherford's absence on war work in 1917. It is interesting evidence of his individuality that, except for one isolated paper written in collaboration with Makower, he continued his spectroscopic work when everyone else at Manchester was absorbed in radioactivity.

In 1920 he was appointed to the chair in physics at the new University College of Swansea. Temporary accommodation was found for the department at the Technical College while the new laboratories were designed and built. The architect has spoken highly of the assistance which he received from Evans in the planning of the laboratories, which were opened in 1922. Installed in these new laboratories, he devoted his energies to building up a school of physics. He laid great stress in training students in the methods of research, and a high percentage of his old students are now filling positions of responsibility. His major interest still remained in the field of spectroscopy, and he instigated work in magneto-optics and in measurements on the extreme ultra-violet. In addition, as was very fitting in a metallurgical centre, a considerable amount of work was done on the physical properties of alloys. In training research students he impressed upon them the importance of accuracy of measurement. He was to be seen at all times of the day examining a photographic plate and suggesting, to some student, other measurements to be made.

The teaching in the department was not neglected. He was an excellent lecturer, very clear and never flustered. He never made the mistake of trying to crowd too much into the time available. His students remember with affection his powerful voice, which was in keeping with his massive build. He was always interested in his students and often surprised his staff by the intimate personal knowledge he seemed to possess of even the most junior student. To his staff he was always considerate and very helpful.

Recently he became absorbed in the post-war problems of the universities, and at the time of his death he was a member of a university committee dealing with reconstruction.

He was a few years short of the retiring age, and in anticipation of this he and his wife had built a bungalow at New Quay in Cardiganshire. It was there he was taken ill in the Christmas vacation, and there he died in July. It is the hope of all who knew E. J. that his wife and family will be comforted in the knowledge that he had gained the affection of all who came in contact with him. L. WRIGHT.

ARTHUR S. NEWMAN

In Arthur S. Newman, who died on 12 August 1943, at the age of 82, the Society lost a member devoted to the design and manufacture of instruments of precision. Like his contemporary R. W. Paul, he passed from scientific instrument making to the design of cinematograph apparatus, but, unlike Paul, he made this his major activity.

Two at least of his inventions in this field have been found of continued value, his film-perforating mechanism, which he himself valued highly, and his pull-down film advance mechanism for cinematograph apparatus. The former represents one aspect of his attitude towards standardization in the motion-picture industry. He was a prominent and untiring advocate of rigid and refined standards in the manufacture and use of both material and apparatus for motion-picture work, from his early days to the time when sound films offered yet more specialized problems.

Photographers will remember Newman particularly as the designer of the early *Sybil* and *Una* still cameras. Among cine cameras his N. and S. Auto-Kine camera was the first spring-driven hand model capable of taking a full magazine of 200 feet without a drop in speed. This camera in its improved form is doing admirable service in many fields today. All his designs are distinguished by his application of sound physical and mechanical principles to the attainment of accuracy and reliability.

Newman made a great personal contribution to the world of technical photography. He held office in the Kinematograph Manufacturers' Association and the British Board of Film Censors, and he helped to found the British Kinematograph Society. He was for a long period chairman of the Kinematograph Section of the Royal Photographic Society, which honoured him with its Progress Medal and an Honorary Fellowship.

Many younger men have reason to be grateful to him for the instruction in craftsmanship and the personal encouragement which they received at his home. It was his custom for many years to keep open house once a week for anyone pursuing photographic interests. Some who knew him would be prepared to maintain that his fostering, in this way, of a spirit of scientific craftsmanship was one of his most valuable and enduring contributions to the advance of photography.

DONALD CLIVE CHALLIS, D.F.M.

Donald Challis was born on 25 May 1913 and received his early education at Colet Court and at St. Paul's School, where he was secretary of the School Engineering Society. On leaving St. Paul's, in 1932, he entered the research laboratories of the electrical-condenser firm of Hellesens as a chemist, and three years later joined Henley's Cables, Ltd. During this period he attended evening classes at Wimbledon Technical College, and in 1940 obtained the B.Sc. degree of London University as an external student. Soon afterwards he joined the R.A.F. and was trained as a pilot in Arizona, U.S.A. ; on completion of his training he returned to England and took part in many night-bombing raids over Germany. In September 1942 he was posted to Africa and participated in operational flights over Italy, and it was during this period that he gained the D.F.M. Although Challis was due for leave in August 1943, he characteristically volunteered for further duty at a critical period in the Allied invasion of Italy. This unselfish gesture cost him his life ; his plane was last seen over the Messina beaches.

Mr. Challis possessed a quiet and generous disposition and a pleasing sense of humour, but those with whom he came into contact will particularly remember his conscientious devotion to any task he had in hand. R. W. B. STEPHENS.

SIR THOMAS RANKEN LYLE, F.R.S.

THOMAS RANKEN LYLE was born at Coleraine, in Northern Ireland, in the year 1860. He died of heart failure on 30 April 1944 at his home in Walsh Street, South Yarra, Melbourne. His father was Hugh Lyle, of " Greenmount ", Coleraine ; his mother, Jane Ranken, of Lisboy.

At the age of 20, Lyle gained a scholarship from the Coleraine Academical Institution which took him to Trinity College, Dublin.

Under the stimulus of the teaching of such men as Fitzgerald, Joly and Townsend, his native ability enabled him to surpass all competitors at the final examinations both in mathematics and theoretical physics and in experimental physics and chemistry. His marks in the final examination were, in fact, the highest on record. His performance elicited enthusiastic testimonials to his ability from his distinguished teachers.

His first teaching post, as a lecturer in mathematics, was at the (Catholic) University in Dublin, and at the same time he served as a representative of Ireland on a commission to inquire into the methods of lighting in lighthouses, thus getting his first acquaintance with problems of technical electricity. In 1889 he applied for, and was elected to, the Chair of Natural Philosophy in the University of Melbourne. This Chair he occupied until, in 1915, he retired, partly for reasons of health and partly owing to the strong persuasion put upon him to undertake other important duties, of which more will be said later.

The duties of a professor of experimental science in an Australian university were, in those days, heavily burdened with teaching work in lecture-room and laboratory. Lyle could not, and did not, shirk his share in these, his only senior assistant being the late Dr. E. F. T. Love. He gave lecture courses to first-, second- and third-year students and special courses to those working for an honours degree. Despite this heavy load, he still found time to carry out research work both in theoretical and experimental physics.

His earlier published papers dealt chiefly with the application of operational (rotating-vector) methods, now familiar to all electricians, to the problems of the alternating-current circuit. Questions of absolute priority are matters for investigation by the historian rather than the biographer, and as regards the credit for the first use of the operator (i or j) in alternating-current theory, the present writer is incompetent to make a pronouncement. Lyle himself says, after referring to the representation of a harmonically varying magnitude as the projection of a revolving vector, " a conception will be introduced which has not, up to the present, been made use of in elementary work. That is, of representing by means of a symbol the operation of turning a line in the positive direction .... through a definite angle in the plane of the diagram ". Lyle then proceeds to exemplify the use of his operator by applying it to the basic types of A.-C. circuit problems.

A much more difficult problem, which Lyle solved many years later (1909) by means of the operational method, is that of the occurrence of harmonics in *both* the armature and the field-coils of the simple alternator. In the ideal case he showed that all odd harmonics occur in the armature, and all even harmonics in the field-coil, a fact which, if generally known, is rarely, if ever, mentioned by writers of text-books on alternating-current machines. This omission may be due rather to the fact that modern alternator design aims at the suppression of harmonics than to the ignorance of their occurrence. At any rate this occurrence and the interactions of field or armature harmonics (and *vice versa*) is basic in the design of the Goldschmidt alternator at Nauen, at one time the most powerful transmitter in the world.

Lyle showed (British Association, 1913) that his theory was competent to give a complete quantitative explanation of the Goldschmidt alternator, the complex operations required in the general case being here simplified by reason of the tuning of the armature and the field-coils to the successive harmonics by means of parallel tuned circuits.

Among Lyle's other important contributions to electrical theory may be mentioned : an elaborate theory of the alternating-current transformer (*Proc. Roy. Soc. Victoria*, 1904) and another on an exact mechanical analogy to the coupled circuits used in wireless telegraphy, etc. (Austr. Assoc. Adv. Sci., 1913). The latter affords another elegant example of Lyle's ingenuity in devising a mathematical method suited to the physical problem.

No account of Lyle as a man would be complete if it were confined merely to the

intellectual side of his nature. His physical activity matched his mental powers. As a student at Dublin he was champion 100-yards sprinter of Ireland, and for four years an international rugby player. " I could out-run them all ", he would say, when speaking of those days. He narrated how, on one occasion, being set against a man in the opposing team notorious for his pugilistic powers—which he exercised freely against his opponents—Lyle picked him up at the first threat of violence and threw him clean over the fence. " I'll fight ye ", said the bruiser, climbing back. " No ", said Lyle, " I won't fight ye, but I'll throw ye over the fence again ".

Lyle's athletic abilities and the interest and support which he gave to athletic sports added to the popularity which he enjoyed both with the students and with colleagues in virtue of his prestige as a scientist and his sound and sympathetic judgments in all matters in which his counsel was sought.

After his retirement from the Chair, Lyle was called upon to fill many important public offices. He was nominated by the Government of Victoria to be the first Chairman of the State Electricity Commission, on which was laid the responsibility for planning the development of the immense brown-coal fields in Gippsland and their use for the generation of electric power and of a distribution system which would link this power-station with other stations, and thus cover the whole State. That this has been successfully accomplished is undoubtedly due not only to Lyle's technical and scientific knowledge but also to his appreciation of the fundamental principles upon which the control and administration of a great industrial organisation must be based—including, as Lyle insisted, freedom from political interference and a humanitarian regard for the welfare of its employees, of which latter aspect a realization is to be seen in the model town of Yallourn at the site of the power-house.

Of Lyle's many other public appointments, the following may be mentioned: Vice-President of the Victorian State Council of Education ; Chairman of the Industrial Exemptions Committee (during the 1914–18 war) ; Chairman of the Standards Association of Australia ; Chairman of the Victorian State Committee of the Council of Scientific Research.

The heavy burden of these civic responsibilities did not quench Lyle's interest and activity in the domain of pure science. Seeking, in this field, a task for his leisure hours, he purchased from Grayson's widow, at a price which showed a fine spirit of generosity, the ruling-machine which Grayson, technician in the Department of Geology, had constructed. He also engaged as an assistant a young mechanic whose liberal wages he paid from his private purse.

He intended to rule diffraction gratings of first-rate quality with the machine, but although many fine gratings were actually ruled—one in the possession of the writer easily resolves the nickel doublet * between the sodium D-lines and shows no " ghosts " even in the fifth order—the result of this venture was, through no fault of Lyle's, a disappointment. He has bequeathed this machine to the National Standards Laboratory (the Australian equivalent of the N.P.L.), and it may be expected soon to achieve all that Lyle had hoped.

Towards the end of 1940 Lyle suffered a cerebral haemorrhage from which he never entirely recovered, though to the very end his mental powers were but slightly impaired and his interest in affairs but slightly diminished. During these years his great regret was that he was unable to contribute to the Empire war-effort, as he had done during the earlier conflict.

In his personal and domestic relationships Lyle's life was a model of integrity, generosity and devotion. All who were privileged to know him—colleagues, students, associates in work and play, family and friends—will remember him with respect, gratitude and affection.

His scientific work was honoured by the doctorate of his *alma mater* and by Fellowship of the Royal Society (1912) ; his public services by Knighthood in 1922.

He is survived by his devoted wife, Lady (Frances Isabel) Clare Lyle, C.B.E. ; a son, Sgt. Thomas Lyle, R.A.A.F. ; and three daughters, Mary (Lady Herring), Nancy and Clare (wife of Lieut.-Cmdr. Brame, R.N.). There are four grandchildren. A brother, Professor Robert P. Ranken Lyle, M.D., of Durham University, and a sister, Mrs. Hanna, of Limavady, Ireland, also survive him.

KERR GRANT.

* This doublet is shown in wavelength tables as a single line, which is interpreted as a doublet in H. N. Russell's analysis of Ni I.

REVIEWS OF BOOKS

Statement on Problems of Scientific and Industrial Research. Pp. 63. (Nuffield College ; Oxford University Press, 1944.) 2*s.* 0*d.* net.

This *Statement* is supported by no less than 92 persons, each of whom is a person of note in science or industry. A few of their names, taken almost haphazard, and arranged antithetically in pairs, will indicate the diversity of personality associated with its general tenor : for example, McGowan and G. D. H. Cole ; H. H. Dale and Harry R. Ricardo ; A. V. Hill and R. E. Stradling ; W. J. Larke and Robert H. Pickard ; Sam Courtauld and Harold J. Laski.

In a document drawn up for approval by such a range of authority, two defects may be expected : the aim may be to indicate the opinions of all, with a consequent lack of clear directive guidance, or the statement may be limited to include only those opinions which are shared by all. Neither of these defects is present to an obvious degree. There is no limitation of the presentation such as might arise in selecting from widely divergent opinions only those which were in accord, nor are any sharp differences of opinion recorded. Yet such differences there must surely have been, had all the signatories attentively scrutinized the array of problems set forth. Is it possible that the desire to speak to the world with unanimity is the reason why the document contains so little evidence of *discussion* ?

Let us consider how a few of the problems are dealt with in this statement, which is mainly limited to the development of industry by application of the sciences of physics and chemistry.

The first problem considered is " How much should be spent on research " ? Various considerations are laid before the reader, but no more definite answer to the question is given than that it should be " much more ". No principle is put forward on which a temperate estimate might be based.

Yet, as regards State expenditure, the Cabinet or the Treasury will be forced to make a decision. All men in the political field whose intentions on the subject are good would have appreciated some guidance on this subject, and what body of people could have been selected better fitted to offer advice than the signatories to this document ?

Certain limits to wise expenditure readily suggest themselves, as, indeed, the authors appreciate. How many physicists and chemists are likely to be free for expansion of the kind in question ? How many can be trained in, say, five years without starving other occupations of intelligent instructors and recruits ? What amount of industry's resources in finance, material and personnel could, in a given time, be devoted to developments arising from research ? These questions are not beyond the range of reasonable conjecture, and the answers would have given some indication of the sum that should be provided.

Under the heading " Waste of ability " the authors conclude that a very high proportion of those possessing an aptitude for industrial research do not fulfil themselves, and that there is thus a wastage of natural ability. Such a view may lead to grave errors of policy. Craftsmen (and women) from the dawn of civilization have used mental processes not unlike those of the scientific man to make the discoveries which form the very framework of civilization. Technical improvements are still being carried out on a vast scale without publication to the world under the cachet of " research ". Are we to divert from industry as much as possible of this talent under the supposition that the ability is at present wasted ? It would be to repeat the error which, a few decades ago, operated to prevent the brilliant boy studying science rather than the Humanities ; or that which in the 1920's and 1930's diverted the most intelligent children away from the workshops into the black-coated occupations.

The importance to individual scientists of change from the atmosphere of fundamental to that of applied science and vice versa is dwelt on (pp. 12 and 32). Doubtless great benefit may be derived from temporary transfer of promising juniors to and from laboratories of commercial firms, universities and research associations, but the suggestion must

be applied with great discretion. Perseverance in a task is no less an education than change of environment.; the young man who enjoys the variety of experience coming from such transfers may, for that very reason, fail to acquire the grit needed to carry through laborious tasks.

While recognizing that the university scientist may render useful service by occasional occupation in industrial problems, the statement points out (p. 16) that the main bulk of applied research must continue to be carried out (*a*) by the State, in its own stations and laboratories, (*b*) by research associations, (*c*) by individual firms. In considering (as also on pp. 30 and 31) which type of laboratory is appropriate for a given investigation, it is pointed out that only a large laboratory can successfully tackle complex problems, and, while the existence of exceptional cases is admitted, we are left with only this guiding principle and the corollaries arising from it, e.g. that only very large firms can expect to derive sufficient return on the expenditure required to maintain an adequate laboratory, and that the small firm faced with a complex problem should supplement its resources for experiment by membership of a research association or State institution.

This sounds impeccably reasonable ; actually it overlooks the basic principles on which decisions concerning the distribution of industrial research should be based. A firm making a product of exceptional quality and reputation may wisely spend on research as much as another, ten times as large, which only aims at turning out a product of fair commercial quality at a competitive price. The question is fundamentally one of finance, and master principles are :—

(*a*) A research should be financed by an organization commensurately with the extent to which the latter is in a special position to exploit any fortunate result arising from it.

(*b*) The expenditure must be limited by (i) the amount of profits (or grants) which the organization can dispose of for the purpose, and (ii) the extent to which the firm can, with its available or obtainable resources, supply the market and make a profit on the products expected to result from the investigation.

In like manner, developments concerning an entire industry should have the researches in connection therewith financed and undertaken by that industry's Research Association, while those researches which concern a number of industries, or in other respects are for the benefit of a large proportion of the nation, should be financed by the State.

A short section considers research in relation to economic and social policy, and envisages the necessity of taking steps to avoid Government research being utilized to build up " monopolist practices " (p. 28). It is a little difficult to see how such a danger would arise. Of more practical importance is the maintenance by the State of standards of quality. That the State has a duty in this respect has been recognized from very early times. In the reign of Elizabeth, in particular, the maintenance of skilled craftsmen was looked upon as one of the main ways of assuring our foreign trade.

This section also lays stress on the need of ensuring that the productive system of the country is related to the needs of consumers in respect of type of product, quality and price ; this is, indeed, regarded as a key problem. That the basic needs of the country must be ensured by State control in the period immediately following the war cannot be gainsaid, but many people consider that if such control should become permanent, it would be a grave danger to our democracy.

Patent laws receive mention, and attention is drawn to the fact that their operation sometimes hampers rather than encourages the setting up of new manufactures. Such cases undoubtedly arise, and at the present time there is a Royal Committee appointed by the House of Commons to consider this very matter. Remedies are, of course, available, although they may entail expensive litigation ; it is an accepted principle that no patent can be maintained if the holder of the rights fails to provide for the public needs. It might be thought that small individual firms could get action taken in appropriate cases through their trade or research associations, but in a recent case which came before the Controller of Patents, it was held that the Association under such circumstances had no *locus standi.*

The often-made proposal that all patents should be laid open to general use on payment

of a reasonable royalty is glanced at but not endorsed. After all, a monopoly (within limits and for a time) is the inducement offered by the State for the introduction of a new process or product ; and any action aimed at weakening the monopoly thereby lessens the inducement.

Everyone will agree with the authors that the State should not give direct financial aid to the research expenditure of private firms beyond what may be granted in the form of tax remissions (p. 39), always provided (as they add) that this should not prevent the State entering on occasion into arrangements with particular firms to undertake special developments necessary in the public interest. The continuance in peace time of the "development contract", which is, in effect, an arrangement of this sort, would have many advantages both for industry and in the public interest.

Research in relation to armaments naturally falls to be dealt with, together with the moot point to what extent the State should conduct its own research in this field. No positive recommendations are made on this highly controversial subject. Having in view that in war today there are few industries which are not playing an essential part, it seems inevitable that so long as private industry exists, private manufacture of armaments (in the widest sense) must remain. An ingenious suggestion is made (p. 42) that there should be a Royal Scientific Volunteer Reserve (R.S.V.R.) of trained scientific and technical men whose services would be available in time of war. Had such a reserve been in existence at the outbreak of the present war, and had it included industrial as well as academic research workers and technicians, it cannot be doubted that we should have got into our stride much earlier. It is strange that no ready means exist of locating industrial research workers. We badly need a list of the industrial research laboratories of Great Britain, similar to that published by the National Research Council of the U.S.A. This publication contains particulars of the staffs and activities of the research laboratories of over 2,000 industrial concerns and professional consultants.

Enough has been said to show what a very wide range of problems find mention in this *Statement.* In applied research the statement of the problem is often halfway to its solution, and perhaps it is scarcely to be expected that at this stage the pros and cons of those problems should be discussed and specific recommendations put forward based on reasoned argument. Yet the present reviewer feels that an opportunity has been lost of proffering solutions with all the enlightenment derived from the recording of diverse conclusions. An *ex cathedra* statement from a bishop cannot be misunderstood—it is *his* opinion ; from a Convocation one desires a complete record of free expression.

The most carping critic, however, cannot deny that the booklet presents a very complete collection of the problems which will arise in extending the provision for scientific and industrial research in this country, an extension which every kind of opinion now advocates. F. TWYMAN.

Physics and Radio, by M. NELKON. Pp. viii + 338. (London: Edward Arnold and Co., 1944.) 8*s*. 6*d*.

This book is primarily intended for boys up to school certificate physics taking the City and Guilds first examination in technical electricity, but will also appeal to a wider circle of radio engineers, who find that their technical knowledge has outstripped their physics. It can be recommended without reserve.

The calculus is avoided and as much information as possible is conveyed graphically by excellent diagrams, of which there are more than 500 in the book. No mathematics beyond linear equations is used, and it is rather surprising to see how far one can go without these aids, generally thought to be essential to physical insight.

The first 150 pages deal with elementary electricity and magnetism ; the treatment of magnetism is rather old-fashioned and might be improved. The vector theory of alternating currents occupies a further 70 pages. A good feature of this section is that some mention is made of non-sinusoidal wave forms. The remaining 140 pages are devoted to radio proper, but are not by any means confined to circuit details or "plumbing". Ample attention is given to the diode, and the theory of detection is explained very clearly ; the triode and the multi-electrode valve are well described and numerous examples illustrate the quantitative considerations which govern the choice of a valve.

The difficult subject of aerials is not shirked. The treatment is based upon an analogy with sound waves, the properties of which are adequately described. The result is excellent, and one is sorry to come to the end of the chapter, which might be expanded in a future edition. The treatment is, of course, entirely qualitative, no mention being made of radiation resistance.

An account of the properties of light based upon Huygen's principle precedes an admirable account of the ionosphere, marred only by a diagram showing the earth and the ionosphere so out of scale as to be misleading. A final chapter deals faithfully with time bases and oscillographs.

The weakest chapter in the radio section is that dealing with oscillators, because the author has tried to include too much and has abandoned his usual practice of devoting most of the chapter to establishing the physical ideas involved on a sufficiently sound basis. In general, more stress could be placed on the concept of energy.

The book contains many numerical worked examples in the text and a good selection of both numerical examples and essay questions at the end of each chapter; by war-time standards it makes a handsome volume, and the price is not excessive when one considers the very large number of diagrams. C. H. C.

The Teaching Profession Today and Tomorrow. A Nuffield College Statement. Pp. 48. (London: Sir Humphrey Milford; Oxford: University Press, 1944.) 1*s.*

It has been estimated that, to implement the new Education Bill, the number of teachers in grant-aided schools—nursery, elementary, secondary, technical and special—will have to be increased from 200,000 to 327,400, assuming that classes in all schools will be reduced to a teachable size. The problem of the supply of these extra teachers is reviewed by the Education Sub-Committee of Nuffield College in a *Statement* published almost simultaneously with the McNair Report.

Even if the Training Colleges were to double their numbers they could not meet the need for 20 years, and new sources of supply must be looked for. There are many men and women in the Services who, although without a secondary education, have shown themselves to be excellent instructors. It is suggested that they should not be excluded from teaching by a rigid and difficult written test. This emergency measure, easier entry to the profession, coupled with easier exit, is recommended as a permanent policy.

The *Statement*, at the same time, emphasizes the importance of educational theory in the training of teachers and suggests that both university graduates and training college students should spend their first year of teaching under supervision and a further two years of teaching and study before taking the Teachers' Certificate and becoming fully qualified.

Discontent among teachers is frankly recognized, and suggestions for improving the status and attractiveness of the profession include smaller classes, the elimination of bad and insanitary buildings, interchange between teachers in schools of different countries and the inspectorate, a sabbatical term with full pay every seven years, and an immediate revision of salary scales. A. E. E. M.

Art and Scientific Thought, by MARTIN JOHNSON. Pp. viii + 192, with 19 half-tone reproductions. (London: Faber and Faber, 1944.) 16*s.* net.

The appearance of this book is symptomatic of the age. Its publication underlines the anxiety which is widely felt that all is not well with the "atmosphere" commonly shared by scientists and artists. Dr. Martin Johnson is exceptionally well placed to undertake a survey, the success of which must depend more upon the sympathy with which it is conducted than upon any formal principles laid down as intellectual dogmas. Writing as a physicist, the author is fully aware of modern tendencies, both theoretical and experimental: at the same time, he has been able to devote considerable attention to the study of art-history and development. In this, he has obviously been helped by Mr. Walter de la Mare, who contributes a pleasing and characteristic foreword.

The main body of the volume consists of a series of essays, dealing with features of resemblance and contrast between the Arts and Sciences, imaginative stimulus, the lack of balance between the Scientific and the Imaginative, and finally, Leonardo da Vinci. Accompanying them are some well chosen illustrations, ranging from examples of Chinese jade to selected Leonardo drawings from the Royal Collection.

Dr. Martin Johnson's thesis amounts to this. We are the inheritors of a vast and complicated network of incompatibilities between the logical and aesthetic in relation to experience. History shows only too clearly how these have arisen, and how devastating have been their effects. An example is provided by the sterilization of Chinese science by an unreasoning and unreasonable conservation, making it impossible for the human mind to exploit its own victories, or even to gain some point of vantage from them. From this, and numerous other instances, it is inferred that there are probably very definite limits beyond which intercourse between laboratory and studio is to the benefit of neither. With this result there should be wide agreement, without in the least detracting from mutual recognition of each other's contributions to the sum total of mankind's experience.

For the scientist, perhaps the most successful sections are those devoted to Leonardo da Vinci. Some of this knowledge is new, and very significant, though, of necessity, conditioned largely by the labours of well known authorities in this field. The influence upon Leonardo of Archimedes stands out, and makes the great artist a master of experimental physics, and that by means of (but not, be it noted, because of overmuch sympathy with) applied mathematics. Dr. Martin Johnson seems to imply that we may thus take Leonardo as a " modern " in scientific outlook. Might it not be better to see in him the prototype of the giants of the nineteenth century rather than of the twentieth ? He had little use for metaphysics ; his genius lay in observation and practice, rather than in the abstractions of theoretical physics, so characteristic of today.

Finally, there is the worship of Natural Law, so overwhelming for Leonardo, and with this veneration a pain in the mind amounting almost to despair at the way mankind abused it. Dr. Martin Johnson brings out this penalty of greatness with a restraint altogether appropriate to its setting. F. IAN G. RAWLINS.

Apprenticeship for a Skilled Trade, by F. TWYMAN; with an Appendix, *Apprentices and the Law*, by H. N. WRIGHT. Pp. iv + 70. (London : Charles Griffin and Co., Ltd., 1944.) 5*s*. net.

The decline in the practice of apprenticeship which has taken place in the last two or three generations has resulted in there existing now but a small appreciation of the major part played by the apprenticeship system in the building and maintaining of British industry. It is therefore of value to be reminded that apprenticeship was at one time the normal and, in fact, the only gateway to the acquirement of that skill in craftsmanship on which the reputation of British industry has so largely rested. Apart from the particular purpose for which Mr. Twyman's book is written, there is in the book much of wide general interest, in that it provides a concise picture of the conditions and purposes of apprenticeship in the early days of the Guilds, of the history of the practice of apprenticeship from those early days until the period of its decline, of the reasons for that decline and of the impact of the humanitarian interest in child labour on the apprenticeship system, and of the present practice of apprenticeship in countries other than our own. The historical and general portions of the book furnish a most interesting background to the main thesis, which is that the skilled trades can only flourish if they are permeated with craftsmanship, and that a renaissance of craftsmanship can best be brought about by a revival of the apprenticeship system.

Mr. Twyman is convinced, and few will disagree with him, that the future prospects of our skilled industries will be much brighter if they include an established apprenticeship system than they could possibly be otherwise, for only thus can the highest craftsmanship be created and maintained. It is assumed in this book that youth and youth's welfare are being, and will be, adequately considered in many quarters, and in consequence Mr. Twyman writes, quite frankly, from an industrial viewpoint. This leads him to express views which diverge sharply from those generally held on the advantages accruing

from the extension of the school-leaving age, if the years of extension are spent mostly in the classroom. A strong plea is made that the early years of an apprenticeship, carried out under suitable indentures which bind employers to make apprenticeship truly instructional, and, by the allowance of free time for attendance at suitable technical institutions, truly educational, and which brings in the local Education Authority as an interested and controlling party, should rank as an extension of the period of a boy's education. These views may not be generally acceptable, but they, and the book which embodies them, are well worth the attention of all those interested in education and in the prosperity of our skilled industries. A. J. P.

RECENT REPORTS AND CATALOGUES

Supplementary List of Publications of the National Bureau of Standards, January 1 1932 to December 31 1941, with Subject and Author Indexes 1901 to 1941. (National Bureau of Standards Supplement to Circular C 24, December 1943.) Pp. 386. U.S. Department of Commerce, Washington, D.C. 50 cents.

Effect of Humidity on Physical Properties of Paper, by F. T. Carson. (National Bureau of Standards Circular C 445, January 1944.) Pp. 12. U.S. Department of Commerce, Washington, D.C. 5 cents.

Studies on Adhesives. Part IX. Tar-Acid-Formaldehyde Resin Adhesives, by D. Narayanamurti and K. Singh. (Indian Forest Leaflet 63, 1944.) Pp. 19. Forest Research Institute, Dehra Dun, U.P. 7*d.*

Microid Balance Desiccator (Leaflet GT 1336) ; *New Super-hard Glass-cutting Knife* (Leaflet GT 1337) ; *Anti-vibration Balance Table* (Leaflet GT 1338) ; *Microid Polishing Alumina* (Leaflet GT 1340) ; *Vibratory Ball Mill* (Leaflet GT 1343). Griffin and Tatlock, Ltd., Kemble Street, London W.C. 2.

ERRATUM

The address of the American Institute of Physics, quoted in recent Advertisements of the *Review of Scientific Instruments* and the *Journal of Applied Physics* as " 173 Fifth Avenue, New York ", is now " 57 East 55th Street, New York 22 ".

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INDEX TO REVIEWS OF BOOKS

10 *November* 1943

The fourteenth meeting of THE COLOUR GROUP, held jointly with the Technical Section of the Paper Makers' Association of Great Britain and Ireland, at the Lighting Service Bureau, Savoy Hill, London W.C. 2, Mr. J. Guild being in the Chair.

A paper on " Colour measurement for control and research in paper making " was presented by S. R. H. Edge, M.A., and Miss H. M. McKenzie, B.Sc., and was followed by an informal discussion.

19 *November* 1943

The tenth meeting of THE OPTICAL GROUP, at the Imperial College, London S.W. 7, Professor A. O. Rankine being in the Chair. A discussion on the " Measurement of aberrations, with special reference to photographic lenses ", was opened by T. Smith, M.A., F.R.S., papers and notes being contributed by H. K. Bourne, J. Home Dickson, J. W. Hasselkus, H. H. Hopkins, L. C. Martin, C. A. Padgham, J. W. Parry, E. W. H. Selwyn and C. G. Wynne.

26 *November* 1943

Science Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

William John Meredith and John Yarwood were elected to Fellowship.

A lecture entitled " A survey of nuclear field theories " was delivered by H. T. Flint, D.Sc., Ph.D., and was followed by an informal discussion.

17 *December* 1943

Science Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

The following were elected to Fellowship : Sydney Eric Ashmore, Willem Hendrik van den Bos, Georgina Maxwell Dadds, Elemer Forbat, Cecil Francis Machin, Noel Percy Mallett, George Ramsay Nicholson, Marcus Laurence Elwin Oliphant, Bertram Samuel Page, Ivor Williams.

It was announced that the Council had elected the following to Student Membership : Peter Andrews, Ivan Bernard Brenton, Derek Edgar Charles Corbridge, Reginald John Corps, R. L. Cowley, James Henry Ellis, Norman Edward Goddard, Robert Louis Gordon, Ralph P. Hudson, Arthur Norman Hunter, Richard Stanley Ince, William Elder Lauder, David Rowland McCall, John Lewis Peake, Derek John Price, Xenia Iris Violet Sweeting, Guy Charles M. R. Tavernier, Kenneth John Veryard, Peter Stephen Williams.

The twentieth Duddell Medal was presented to Mr. John Guild in recognition of his invention, design and development of many optical instruments of outstanding merit.

The following demonstrations were given and were followed by an informal discussion :

" The spinning and floating of light bodies in an electric field : a development of an experiment by Sir Isaac Newton ", by Lord Rayleigh, Sc.D., LL.D., F.R.S. ;

" A machine for harmonic synthesis ", by A. Shilton, B.Sc. ;

" A new method of measuring the inclination of the Earth's magnetic field ", by E. P. Harrison, Ph.D., and Miss E. Hamilton Smith, M.A.

26 *January* 1944

The fifteenth meeting of THE COLOUR GROUP, at the Imperial College, London S.W. 7, Mr. J. Guild being in the Chair.

A paper entitled " Some difficulties encountered in applying the theory of McAdam's limiting brightness to real dyestuffs " was read by T. Vickerstaff, M.Sc., Ph.D., and was followed by an informal discussion.

11 *February* 1944

Science Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

The following were elected to Fellowship : Donald Robert Barber, William Barrow Bartley, Robert Blackadder Fisher, Alexander Rule Ingles, Frederick John Scrase.

It was announced that the Council had elected the following to Student Membership : Robert Bernard Clark, John Francis Irving Cole, Geoffrey Harvey Haggis, R. G. Lingard, Oliver Simpson, Ian Murray Watt.

An informal discussion on " Band spectra and energies of dissociation of diatomic molecules " was held, the contributors being R. F. Barrow, D.Phil., A. G. Gaydon, D.Sc., W. Jevons, D.Sc., R. C. Pankhurst, Ph.D., and R. W. B. Pearse, D.Sc.

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18 *February* 1944

The eleventh meeting of THE OPTICAL GROUP, at the Northampton Polytechnic, London E.C. 1, Professor A. O. Rankine being in the Chair.

The following papers were read and discussed :

" Some notes on space perception ", by H. H. Emsley.

" Contact lenses ", by K. Clifford Hall.

" A note on the treatment of a case of conical cornea by contact lens ", by E. F. Fincham.

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3 *March* 1944

Science Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

The following were elected to Fellowship : Francis Henry Aldred*, James Graham Ballantyne*, Kathleen Virtue Ballington*, Kenneth Denny Barritt*, Ronald Percy Bell, Robert Berman*, John Weston Broadhurst*, Brian Harvey Chirgwin*, Maurice John Daintith*, Ernest William Emery*, James Mackenzie Falconer*, Ivan Oscar Fieldgate, Ernest John French*, Eileen Elsie Fuller*, Anthony James Hailwood, David Arthur Jones, Helen Dick Megaw, Derwent Mark Atkinson Mercer, Hans Oskar Puls, Richard Francis Yeoman Randall*, George Codling Richer, Dennis Parker Riley, William Raymond Shackcloth*, Harold A. B. Simons*, Thomas Anderson Straughan*, Robert Soulsby Tebble*, Derrick Michael Theaker, James Topping, Stephen Edelston Toulmin*, Philip Robert Tunnicliffe*, William Wilson, Malcolm Frank Wintle*, Peter Osborn Wymer*.

(* Transferred from Student Membership.)

It was announced that the Council had elected the following to Student Membership : Robert Guy Chambers, Ronald Henry Hall, Peter T. Landsberg, Denis Noel Layton, Martin Ludwig Loehr, Derek Hubert Mash.

An address on " Physics and the scientific instrument industry " was delivered by A. J. Philpot, O.B.E., M.A., B.Sc., and was followed by a discussion opened by F. Twyman, F.R.S.

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8 *March* 1944

The fifteenth meeting of THE COLOUR GROUP, at the Imperial College, London S.W. 7, Mr. G. S. Fawcett being in the Chair.

A paper entitled " The Sub-Committee on Colour Deficiency in Industry : A progress report " was read by J. H. Shaxby, D.Sc.

The meeting was preceded by the fourth *Annual General Meeting of the Colour Group* for the presentation of the Committee's report on the work of the Group in 1943–44 and the election of Officers and Committee for 1944–45.

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15 *March* 1944

Science Meeting, held jointly with the Plastics Group of the Society of Chemical Industry, at the London School of Hygiene and Tropical Medicine, London W.C. 1, the President, Professor E. N. da C. Andrade, being in the Chair.

A lecture on " X-ray examination of plastics " was delivered by W. T. Astbury, Sc.D., F.R.S., and was followed by an informal discussion.

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21 *April* 1944

The twelfth meeting of THE OPTICAL GROUP, at the Imperial College, London S.W. 7, Professor A. O. Rankine being in the Chair.

A lecture entitled " My fifty years in the optical industry " was delivered by J. W. Hasselkus, C.B.E.

The meeting was preceded by the third *Annual General Meeting of the Optical Group* for the presentation of the Committee's report on the work of the Group in 1943–44 and the election of Officers and Committee for 1944–45.

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26 *April* 1944

Science Meeting, at the Royal Institution, London W. 1, the President, Professor E. N. da C. Andrade, being in the Chair.

The following were elected to Fellowship, the first three being transferred from Student Membership: Bernard Cyril Abbott, Patricia Mary Lord, Henry Vernon Walters, Hans Samuel Joachim, Eric Balliol Moullin, Oliver Graham Sutton.

It was announced that the Council had elected the following to Student Membership: Stanley Gill, Edward Peter Campbell Sington, Colin Henry Smith, Peter Anthony Tanner.

The twenty-eighth Guthrie Lecture was delivered by Professor J. H. Hildebrand, Sc.D., Ph.D., who took as his subject " The liquid state ".

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29 *April* 1944

Science Meeting, at the new Clarendon Laboratory, Oxford, the President, Professor E. N. da C. Andrade, being in the Chair.

Professor Hildebrand repeated the twenty-eighth Guthrie Lecture.

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24 *May* 1944

Annual General Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

The minutes of the previous Annual General Meeting were read and accepted as correct.

The reports of the Council and the Honorary Treasurer and the Annual Accounts for 1943 were adopted.

The Officers and Council and the Auditors for 1944–45 were elected.

Votes of thanks were accorded to the Rector and Governing Body of the Imperial College, the Managers of the Royal Institution, the Director of the Science Museum, and the Electric Lamp Manufacturers' Association for excellent accommodation at meetings, to the retiring Officers and Council, and to Professor A. F. C. Pollard for preparing the U.D.C. Index Slips for the *Proceedings*.

The meeting was followed by an *Extraordinary General Meeting* for the election of Academician Abram Feodorovich Joffé as an Honorary Fellow of the Society.

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24 *May* 1944

Science Meeting, at the Imperial College, London S.W. 7, the President, Professor E. N. da C. Andrade, being in the Chair.

The following were elected to Fellowship : John Iball, Robert Elias Lloyd-Owen, Keith Burton Logie (transferred from Student Membership), Albert Lloyd George Rees, Bhrigunath Narayan Singh, Douglas Harold Tomlin.

It was announced that the Council had elected the following to Student Membership : Deryck Russell Bell, David Russell Bland, Peter Albert Michaels, R. V. Waterhouse.

A lecture on " Current problems in visual research " was delivered by W. S. Stiles, D.Sc., Ph.D., and was followed by an informal discussion.

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30 *May* 1944

A special meeting of The Optical Group, held jointly with the Scientific and Technical Group of the Royal Photographic Society, at 16 Prince's Gate, London S.W. 7 Professor A. O. Rankine being in the Chair.

E. R. Davies, B.Sc., presented for informal discussion an address entitled " Psychophysics and photography ", by Dr. Loyd A. Jones of the Kodak Research Laboratories, Rochester, N.Y.

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7 *June* 1944

The seventeenth meeting of The Colour Group, at the Royal Society of Arts, Adelphi, London W.C. 2, Mr. J. Guild being in the Chair.

A paper on " The Munsell System : a review of recent work carried out by a sub-committee of the Optical Society of America on the spacing of the Munsell Colours " was read by W. D. Wright, D.Sc., and was followed by an informal discussion.

REPORT OF COUNCIL FOR THE YEAR ENDED 31 DECEMBER 1943

INTRODUCTORY AND GENERAL

In spite of ever-tightening war-time restrictions and difficulties, the Society carried on its work with hardly any slackening of pace throughout the year now under review. The Science Meetings were satisfactorily attended ; the usual six Parts of the *Proceedings* appeared with but slight delays ; the ninth volume of the *Reports on Progress in Physics* was published towards the end of the year and is in very wide request ; the demand for earlier volumes of the *Reports* and for other publications has been very well maintained ; and the Colour and Optical Groups have again been conspicuously successful. It is gratifying to report a further growth of the membership, which stood well over 1300 at the end of the year and has continued to rise since then.

Although the costs of paper, of printing (the rates for which are now 54 per cent higher than in June 1939), and of all essential services have increased steadily during these years of war, no change has been made in the rates of Fellows' and Student Members' annual subscriptions, apart from the revision, foreshadowed in last year's Report of Council, of the scheme of reduced conjoint subscriptions payable through the Institute of Physics. In accordance with this revision the discount has been diminished from 15, 25 or $33\frac{1}{3}$ per cent to a uniform 10 per cent in each case of Fellowship of two or more participating Societies, and it is satisfactory, though not surprising, that no word of complaint has reached the Officers and Council.

The situation immediately after the war is clearly one that demands the most careful consideration by the Officers and Council. Fellows are invited to co-operate by making a special and immediate effort to bring the advantages of Fellowship of the Society to the notice of their colleagues and the great privileges of Student Membership to their senior physics students. There is little doubt that the recent rate of increase of membership will be one of the factors taken into account when the future housing of learned societies comes under consideration. The Society's position was briefly stated by the President in the following letter, which appeared in *The Times* on 22 December 1943 :—

To the Editor of 'The Times'.

Sir : In his presidential address to the Royal Society, in which he dealt with the accommodation of the Royal Society and of the scientific societies in general, Sir Henry Dale said that, while the authorities of the 1870s did not see much future for chemistry, apparently they did not foresee any future for physics at all. No doubt he had in mind the Physical Society, which is at present housed, by courtesy of the Royal Commissioners for the Exhibition of 1851, in the top floors of what was a private house in Kensington, to which the only access is a forbidding flight of stairs. The Society is very grateful to the Royal Commissioners, who have always shown great kindness and consideration ; without this help the Society would be in grave difficulties. At the same time, in view of the services which the physicists of the country have rendered to the State, it may be contended that it has deserved some worthier accommodation.

The Society, which was founded in 1874, has increased its membership from about 100 in the early days to some 1350 at the moment, and it is growing rapidly. Among its past presidents it numbers Lord Kelvin, Balfour Stewart, G. F. Fitzgerald, Oliver Lodge, Arthur Schuster, J. J. Thomson, W. H. Bragg, Sir Arthur Eddington, Lord Rayleigh and Sir Charles Darwin. It would like to have a home of its own in a central position, near its great ancestor the Royal Society, and hopes that its demure virtues will not be neglected when the general question raised by Sir Henry Dale comes up for serious consideration.

I am, Sir,

E. N. da C. Andrade,
President, Physical Society.

1 Lowther Gardens,
Exhibition Road S.W. 7
20 December 1943.

The Council wishes to record its thanks for hospitality extended to the Society and the two Groups during the year by the Rector and Governing Body of the Imperial College, by the Managers of the Royal Institution, by the Electric Lamp Manufacturers' Association, by the Director of the Science Museum, and by the Director of the National Institute for Medical Research. The thanks of the Society are also accorded to Professor A. F. C. Pollard for his continued expert help in the preparation of the Universal Decimal Classification Index Slips, which are supplied with the *Proceedings* to all who need them.

MEETINGS

An Annual General Meeting was held at the Royal Institution on 18 May 1943 for the presentation of the 1942 Reports of the Council and the Honorary Treasurer and for the election of the Officers and Council for 1943–44.

Nine Science meetings were held during the year : five at the Imperial College, three at the Royal Institution, and one at the Science Museum. Five of them were devoted to the special lectures reported in the following paragraphs, and two to " lecture-surveys " by Dr. Bruce Chalmers on " The Non-destructive Testing of Metallic Components " (*Proceedings*, **56**, 132 (1944)) and by Dr. H. T. Flint on " A Survey of Nuclear Field Theories " (*Proceedings*, **56**, 149 (1944)). At one meeting, exhibits and demonstrations illustrative of the radio instruction in the City and Guilds Electrical Engineering Department of the Imperial College were arranged and described by Professor C. L. Fortescue and his staff. The remaining meeting was for demonstrations by Dr. E. P. Harrison and Miss E. Hamilton-Smith, by Lord Rayleigh, and by Mr. A. Shilton (*Proceedings*, **56**, 31, 73 and 130 (1944)).

GUTHRIE LECTURE

The twenty-seventh Guthrie Lecture was delivered by Professor E. T. Whittaker on 18 May at the Royal Institution, the subject being " Chance, Freewill and Necessity in the Scientific Conception of the Universe " (*Proceedings*, **55**, 459 (1943)).

THOMAS YOUNG ORATION

The thirteenth Thomas Young Oration was delivered on 4 June 1943 at the Science Museum by Professor F. C. Bartlett, who took as his subject " Some Current Problems in Visual Function and Visual Perception " (*Proceedings*, **55**, 417 (1943)).

CHARLES CHREE MEDAL, PRIZE AND ADDRESS

The second award of the Charles Chree Medal and Prize was made by the Council to Professor (now Brigadier) B. F. J. Schonland in recognition of his researches on atmospheric electricity and the phenomena of the lightning stroke. The presentation was made at the Royal Institution on 16 July 1943, when Brigadier Schonland delivered the second Charles Chree Address, which was entitled " Thunderstorms and their Electrical Effects " (*Proceedings*, **55**, 445 (1943)).

DUDDELL MEDAL

The award of the nineteenth Duddell Medal to Dr. C. R. Burch was announced in last year's Report of Council. The presentation took place at the Imperial College on 9 April 1943, and was followed by an address by the Medallist, under the title " A Technologist looks at the Future ", an abridged account of which was published later (*Nature*, **152**, 523 (6 November 1943)) and supplied to each member of the Society.

The Council awarded the twentieth Duddell Medal to Mr. John Guild for his invention, design and development of many optical instruments of outstanding merit. The presentation was made at the Imperial College on 17 December 1943, and a note on the Medallist and his work has been published in the *Proceedings* (frontispiece to vol. 56, January 1944).

PARSONS MEMORIAL LECTURE

In accordance with the terms governing the appointment of the Parsons Memorial Lecturers, the eighth annual lecture of the series was given under the auspices of the Physical Society. It was delivered at the Royal Institution on 15 October 1943 by the Right Hon. Lord Rayleigh, who took as his subject " Optical Topics, in part connected with Charles Parsons " ; it was later published *in extenso* (*Nature*, **152**, 676 (11 December 1943)), and copies were supplied to all members of the Society.

PROCEEDINGS

Volume **55** (1943) of the *Proceedings*, which was published in the usual six two-monthly parts, is only slightly smaller than the 1942 volume. A slight increase in the supply of paper made it possible to reprint one part of Volume **54**, no copies of which remained in stock, and to print more copies of Volume **55**. The charges for advertisements were increased during the year, the increases taking effect at the next renewal of each contract.

REPORTS ON PROGRESS IN PHYSICS

Volume **9** (1942–43) of the *Reports* was published in December, and has been in very great demand both inside and outside the Society, in this country, in the Empire and in the U.S.A. The use of two years' supply of paper for a single volume has made possible the printing of a larger number than in the case of previous volumes. The remaining stock of Volume **3** (1936) became exhausted towards the end of the year, and the sales of the other available volumes, **5** (1938) to **8** (1941), are still highly satisfactory.

Through the kindness of Miss M. M. Mitchell, Publications Manager of the American Institute of Physics, to whom the Council again records its thanks, American physicists still obtain copies of the *Reports* directly from the office of the Institute, 57 East 55th Street, New York, 22. The Editor of the *Reports*, Dr. W. B. Mann, as the guest of honour at the recent dinner of the American Physical Society in New York, gave oral expression of the Society's gratitude for this valuable co-operation.

REPRESENTATION ON OTHER BODIES

The following were the Society's representatives on other Bodies in 1943 :—

On the National Committee for Physics : Dr. Ezer Griffiths, Prof. N. F. Mott and Mr. R. S. Whipple.

On the National Committee for Scientific Radio : Mr. J. A. Ratcliffe and Sir Robert Watson Watt.

On the Committee of Management of Science Abstracts : Mr. J. H. Awbery, Prof. Allan Ferguson, Dr. W. Jevons and Dr. D. Owen.

On the Board of the Institute of Physics : Dr. W. Jevons and Prof. A. O. Rankine.

On the Council of the British Society for International Bibliography : Prof. L. C. Martin.

At the Inauguration of the English Scientific Films Association : Mr. W. Ashhurst and Mr. F. F. Renwick.

MEMBERSHIP

A limited edition of the List of Officers and Members, the only one issued since 1938, was printed in the latter part of the year. As is shown in the following table, the total membership increased during the year by about 6 per cent, the Fellowship increasing by 3·4 per cent and the Student Membership by 16·8 per cent. Although it is pleasant to record a considerable growth for the third year in succession and the passing of the thousand mark in the Fellowship and of the thirteenth hundred mark in the total membership, the figures are lower than might reasonably be expected after the remarkably fine record for 1942.

Roll of Membership	Hon. Fellows	Hon. Fellows, Optical Society	Ex-officio Fellows	Fellows	Student Members	Total
Totals, 31 Dec. 1942	10	3	4	998	232	1247
Changes during 1943: Newly elected	1			42 } 64	74	
Transferred				22	22	
Deceased	1			15	1	
Resigned				8 } 30	3 } 35	
Lapsed				6		
Suspended				1	9	
Net increase				34	39	73
Totals, 31 Dec. 1943	10	3	4	1032	271	1320

OBITUARY

The Council records with regret the deaths of the following Fellows :—Mr. R. M. Abraham, Mr. G. L. Addenbrooke, Mr. Rollo Appleyard, Mr. D. C. Challis, Major Leonard Darwin, Lord Hirst of Witton, Mr. A. S. Newman, Mr. R. W. Paul, Mr. F. F. Renwick, Dr. Alexander Russell (Past President), Mr. P. A. Scott-Iversen, Mr. W. O. Smith, Mr. H. Dennis Taylor, Mr. T. B. Vinycomb, Dr. F. J. W. Whipple, Professor P. Zeeman (Honorary Fellow), and the death of a Student Member, Mr. D. H. T. Gant.

COLOUR GROUP

The third Annual General Meeting of the Group was held at the Science Museum on 3 March 1943, when Mr. J. Guild was elected as Chairman, Mr. H. D. Murray was re-elected as Honorary Secretary and the Committee for 1943–44 was elected.

Four Science Meetings of the Group were held : on 3 March and 9 September at the Science Museum, and on 12 May and 10 November at the Lighting Service Bureau of the Electric Lamp Manufacturers' Association, the last of these being held jointly with the Technical Section of the Paper Makers' Association. The subjects of the Meetings were " The Significance and Limitations of the C.I.E. Standard Observer Tables " (address by Mr. J. Guild), " Sixty Years of Colorimetry " (lecture by Mr. G. S. Fawcett, *Proceedings*, **56,** 8 (1944)), " A Review of the Theory of Colour Photography " (lecture by Mr. J. B. Reid), " Colour Measurement for Control and Research in Paper-making " (paper by Mr. S. R. H. Edge and Miss H. M. McKenzie, *Proc. Tech. Sec. P.M.A.* **24,** 201 (1943)). The attendances averaged about 40.

The work of the Sub-Committees on Colour Blindness in Industry and Colour Terminology continued throughout the year.

The membership of the Group increased slightly during the year, and on 31 December was as follows :

Members of the Physical Society	69
Members of other participating bodies	70
Members of six firms subscribing for sustaining membership	15
Other members	3
Total ..	157

OPTICAL GROUP

At the second Annual General Meeting of the Group, which was held at the Imperial College on 16 April 1943, Professor A. O. Rankine and Professor L. C. Martin were re-elected as Chairman and Honorary Secretary, respectively, and the Committee for 1943–44 was elected.

During the year five Science Meetings of the Group were held: four of them were at the Imperial College on 22 January, 16 April, 18 June and 19 November, the subjects discussed being "The Resolution of Optical Images" (papers by Prof. L. C. Martin, Mr. H. H. Hopkins and Dr. K. B. E. Merling, *Proceedings*, **55**, 104 and 116 (1943); **56**, 321 (1944)), "The Limit of Visual Resolution" (paper by Mr. E. W. H. Selwyn, *Proceedings*, **56**, 286 (1944)), "The Designing of Systems with Non-spherical Surfaces" (discussion opened by Instr.-Capt. T. Y. Baker, *Proceedings*, **56**, 481 (1944)), "Optical Aids to Teaching" (lecture by Mr. H. E. Dance, *School Science Review*, **25**, 172 (1944)) and "The Measurement of Aberrations, with special reference to Photographic Lenses" (discussion opened by Mr. T. Smith, *Proceedings*, **56**, 277 (1944)). A meeting was held at the National Institute for Medical Research, Hampstead, on 17 September for a demonstration of the electron microscope and other apparatus. Members of the Group joined the Manchester and District Branch of the Institute of Physics on 20 March for a demonstration of the electron microscope at Shirley Institute, Didsbury.

The membership of the Group increased by 33 during the year, and on 31 December was as follows:

Members of the Physical Society	133
Members of other participating bodies	111
Members of twenty-two firms subscribing for sustaining membership	83
Other members	9
Total	336

REPORT OF THE HONORARY TREASURER FOR THE YEAR ENDED 31 DECEMBER 1943

During 1943 income exceeded expenditure by about £578, as compared with £649 in the previous year, the total expenditure (£4485) being roughly £444 more than in 1942 and the total receipts (£5063) being £373 more. The additional expenditure was mainly on publications: there was a general increase in printers' charges, we published a new volume of *Reports on Progress in Physics* (no volume of which was printed in 1942), and we reprinted one number of Vol. **54** of the *Proceedings*, which was out of print. It is satisfactory to note, however, that the claim on the Society for the cost of *Science Abstracts* was considerably less and the return slightly better than in 1942, and that the receipts from annual subscriptions of Fellows and Student Members and from sales of publications were all higher.

The Society's holding of £500 of India $3\frac{1}{2}\%$ Stock was redeemed at par at the beginning of the year, this being the only change in the Society's investments. The total market value of the remaining investments at the end of 1943 was about £127 less than a year earlier.

A further publications grant of £200 by the Royal Society from the Rockefeller Foundation Gift is gratefully acknowledged. In due course this will be expended on the reprinting of further numbers of the 1942 *Proceeedings*, to make up the deficiency to which attention was drawn in last year's reports of the Council and the Treasurer.

(Signed) CLIFFORD C. PATERSON,
Honorary Treasurer.

26 *April* 1944.

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31 DECEMBER 1943

1942 £	Expenditure	£ s. d.	£ s. d.
	To		
320	" Science Abstracts "		265 15 1
	Normal Publications :		
1513	" Proceedings "	1687 6 1	
47	Agenda Papers and Notices	66 15 0	
834	" Reports on Progress in Physics "	1119 7 7	
			2873 8 8
254	Postage on Publications and General Correspondence		266 17 10
64	Expenses at Meetings		53 4 7
72	Honoraria to Special Lecturers		40 0 0
	Administration Expenses :		
850	Secretarial and Clerical Assistance and Office Expenses	933 13 9	
87	1 Lowther Gardens (including Air Raid Precautions)	52 7 6	
			986 1 3
649	*Balance, being excess of Income over Expenditure, carried forward to Accumulated Fund*		577 14 3
£4690			£5063 1 8

1942 £	Income	£ s. d.	£ s. d.
	By		
	Subscriptions :		
57	Entrance Fees	38 17 0	
1757	Fellows	1920 3 3	
123	Student Members	140 3 6	
24	Colour Group	25 2 6	
57	Optical Group	58 4 0	
114	For " Science Abstracts " and Advance Proofs	139 10 1	
			2322 0 4
	Sales :		
764	" Proceedings "	884 6 2	
863	" Reports on Progress in Physics "	902 17 5	
105	Special Publications	133 14 0	
			1920 17 7
332	Advertisements in " Proceedings "		306 18 8
284	Dividends from Investments		283 5 1
10	Transfer from Life Composition Fund of amounts paid by Fellows now deceased		30 0 0
200	Publications Grant by the Royal Society from the Rockefeller Foundation Gift		200 0 0
£4690			£5063 1 8

BALANCE SHEET AS ON 31 DECEMBER 1943

Liabilities

	£	s.	d.	£	s.	d.
Sundry Creditors				217	18	1
Life Compositions:						
As on 31 December 1942	1534	0	0			
Add Payments during year	105	0	0			
	1639	0	0			
Less Transfer to Income and Expenditure Account	30	0	0			
				1609	0	0
Subscriptions received in advance:						
Members	23	1	0			
Publications	176	11	0			
				199	12	0
Special Funds:						
W. F. Stanley Trust Fund	259	0	0			
Duddell Memorial Trust Fund	337	14	9			
"Progress Reports" Reserve Account	83	1	0			
Herbert Spencer Legacy	239	7	6			
Charles Chree Medal and Prize Fund:	1880	5	10			
				2799	9	1
General Fund:						
As on 31 December 1942	4995	12	3			
Add Balance of Income and Expenditure Account	577	14	3			
„ Profit on India Stock redemption	88	0	0			
				5661	6	6
				£10487	5	8

Assets

	£	s.	d.	£	s.	d.
Investments at Market Value on 31 December 1939:						
W. F. Stanley Trust Fund	259	0	0			
Duddell Memorial Fund	374	0	0			
Charles Chree Medal and Prize Fund	1866	10	0			
General Fund	5505	10	0			
				8005	0	0
(Market Value on 31 December 1943: £9173)						
Dividends due from Investments				50	4	7
Inland Revenue—Income Tax recoverable for 1942 and 1943				109	19	1
Subscriptions due				40	16	10
Sundry Debtors				377	16	6
Stock of Paper and Binding Material				67	9	4
Cash at Bank	484	16	5			
„ *in Post Office Savings Bank Account*	1351	2	11			
				1835	19	4
				£10487	5	8

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the bank balances and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

Spencer House, South Place, E.C. 2.
21st April 1944.

KNOX, CROPPER & Co.,
Chartered Accountants.

LIFE COMPOSITION FUND ON 31 DECEMBER 1943

	£	s.	d.
25 Fellows paid £10	250	0	0
1 Fellow paid £10 10s.	10	10	0
1 Fellow paid £15	15	0	0
14 Fellows paid £21	294	0	0
33 Fellows paid £31 10s.	1039	10	0
	£1609	0	0

SPECIAL FUNDS

W. F. STANLEY TRUST FUND

	£	s.	d.		£
Carried to Balance Sheet	259	0	0	£300 Southern Railway Preferred Ordinary Stock	199
				£442 Southern Railway Deferred Ordinary Stock	60
	£259	0	0		£259

DUDDELL MEMORIAL TRUST FUND

Capital

	£	s.	d.		£
Carried to Balance Sheet	374	0	0	£400 3½% War Loan Inscribed "B" Account	374

Revenue

	£	s.	d.		£
Balance on 31 December 1942	4	18	9	Interest on War Loan	14
Honoraria	40	0	0	Balance carried to Balance Sheet	36
Medals and Certificates	5	6	6		
	£50	5	3		£50

"PROGRESS REPORTS" RESERVE ACCOUNT

	£	s.	d.		£
Balance carried to Balance Sheet	83	1	0	Balance on 31 December 1942	83

HERBERT SPENCER LEGACY

	£	s.	d.		£
Balance carried to Balance Sheet	239	7	6	Balance on 31 December 1942	239

CHARLES CHREE MEDAL AND PRIZE FUND

Capital

	£	s.	d.		£
Balance carried to Balance Sheet	1865	16	4	Balance on 31 December 1942	1865

Revenue

	£	s.	d.		£
Charles Chree Prize	131	5	0	Balance on 31 December 1942	83
Medal and Certificate	6	15	6	Interest on Investments	68
Balance carried to Balance Sheet	14	9	6		
	£152	10	0		£152